Dynamic Competition, Innovation and Strategic Financing

by

Matthew Spiegel†

and

Heather Tookes‡

Yale School of Management

P.O. Box 208200

New Haven, CT 06540-8200

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† Email: matthew.spiegel@yale.edu, phone (203) 432-6017, fax (203) 436-0630, http://som.yale.edu/~spiegel.
‡ Email: heather.tookes@yale.edu, phone (203) 436-0785, fax (203) 436-0630, http://www.som.yale.edu/faculty/het7/.

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ABSTRACT

This paper models the interactions among product market innovation, product market competition, and corporate financing decisions in the context of a dynamic duopoly. One competitor faces an opportunity to adopt a new technology. If adopted, the firm must also determine whether it will obtain public or private financing. Our results allow us to relate current firm and industry characteristics to these decision variables. In particular, larger, more profitable firms with small rivals have the greatest incentives to innovate. The private versus public financing decision depends mainly on the magnitude of the technological improvement and length of the period during which private financing extends the innovator's product market advantage. Due to the model's formulation it is both tractable and amenable to empirical estimation. We estimate the model and provide estimates of the value of innovation and private financing for a sample of industries and firms.

How important are the interactions among innovation, product market competition and the going public decision?[1] This paper presents a tractable framework for examining financial decision making in dynamic competitive environments. In particular, it derives explicit, closed form solutions for a dynamic duopoly in which a firm decides whether to adopt an innovation. If adopted, the firm must determine whether it will obtain public or private financing. The model allows one to relate current firm and industry characteristics to these decision variables in ways that are empirically measurable.

The potential competitive benefits of private financing can be seen in any industry in which there is value associated with keeping competitive secrets. Consider Management's discussion of the advantages of initial growth as a private firm in Google's S-1 filing (for its 2004 IPO): "As a smaller private company, Google kept business information closely held, and we believe this helped us against competitors." The actual importance of competitive structure in financing decisions is an empirical question, and an important advantage of this paper is that it provides closed form solutions and several testable hypotheses that can easily be matched with available data. Furthermore, because the model takes place in real time its forecasts can be tested quantitatively as well as qualitatively. The former is typically very difficult to accomplish when testing two-period models. While other directly testable dynamic models are relatively rare in the capital structure literature notable exceptions include Leland (1994), Leland (1998), Goldstein, Ju and Leland (2001), and Hennessey and Whited (2005, 2007).[2]

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[1] In the results of their survey of CFOs, Brau and Fawcett (2006) report that "Disclosing information to competitors" and "SEC reporting requirements" ranked fourth and fifth, respectively, of 11 factors contributing to the firms' decisions to undertake an IPO. Not surprisingly, these factors were most important to firms actually undertaking the IPO as well as those that were large enough to go public but did not undertake an IPO. They were less important to firms with withdrawn IPOs.

[2] For example, Welch (2001, p. 11) writes: "Can we build a good model of, e.g., optimal corporate leverage decisions based on observable firm characteristics, and measure the effects of moving towards/away from this optimum?...I dream of models whose predictions are more quantitative than qualitative; models which are of direct use to empiricists."

Potential issuers face a tradeoff between releasing information and the promised return when deciding between a public or private financing. Public issuance in the United States involves the release of information that is potentially valuable to competitors and thus may hurt the future product market performance of the issuer.[3] On the other hand, private financing involves a limited number of investors who may require higher returns due to, for example, the relative illiquidity of their investment. Indeed, there is a well documented discount for private securities (see e.g., Hertzel and Smith (1993) for an analysis of private placements).

To address these issues, we solve a differential game based upon a variant of the Lanchester (1916) "battle" model. In our application two firms compete against each other for market share by spending funds to acquire each others customers. The adaptation we develop provides a simple, yet flexible structure for examining the dynamic interactions among product market competition, innovation and public versus private financing.

In the model, one firm has an opportunity to invest in a new value enhancing technology. Assuming the firm decides to innovate it then chooses between financing development costs with public or privately placed securities.[4] The main benefit of private financing is that it allows the firm to extend the time during which it can hide its technological progress. This in turn extends

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[3] The competitive effects of information-sharing are well-known in the industrial organization literature. Vives (1990) provides a survey. In general, sharing of cost information occurs in equilibrium under Cournot competition and does not occur under Bertrand (the results are the reverse for common value demand information). See also Li (1985) and Gal-Or (1985). There is a related accounting literature on disclosure in imperfectly competitive markets e.g., Darrough and Stoughton (1990); Wagenhofer (1990); Masako (1993)). With the exception of Bhattacharya and Ritter (1983) and the extension to debt markets in Bhattacharya and Chiesa (1995), in which a firm with private information takes into account both the impact of disclosure on the ability to raise funds in financial markets and the probability of the success of a rival firm in an R&D game, these issues have received little attention in the finance literature. Note, also, that these papers do not model dynamic competitive interactions.

[4] While they do not focus on competitive interactions, there has been recent empirical research examining firms' decisions to obtain private versus public financing (see e.g., Bharath and Dittmar (2007) and Gomes and Phillips (2007).

the time it retains its competitive advantage over its rival, which eventually succeeds at copying the innovation.[5]

The model's duopoly setting allows it to produce predictions regarding the competitive environment's impact on innovation and financing decisions; insights that are obviously impossible to derive in a single firm model. It is also possible to examine how the type of innovation influences a firm's financial and marketing decisions. Profits in the model depend both upon the relative ability of each firm to lure away its rival's customers, and the profit per unit of market share the firm obtains from those customers. Consider, for example, an innovation of the former type that makes it easier for a firm to attract market share. In this case, the model finds that increasing a rival's ability to attract customers can encourage innovators to use public financing. Many other questions like these can be addressed as well, such as the impact of technological innovations and financing decisions by one firm on the value of another.

The paper's results indicate that the relative size of the firms within an industry plays an important role in the innovation decision. In particular, larger, more profitable firms with small rivals have the greatest incentives to innovate. Intuitively, this is because small, less profitable firms are less able to withstand the aggressive competitive behavior by rivals that their own innovation triggers. The positive association between firm size and the number of innovations predicted by the model has been documented empirically (see e.g., Acs and Audretsch (1988), Henderson and Cockburn (1996), and Nahm (2001)).

Innovations that increase a firm's profits per unit of market share lead to dynamics that differ in fundamental ways from those that increase the attractiveness of its product. Once it becomes public knowledge that a firm has an innovation of the former type, then all firms in the

[5]Maksimovic and Pichler (2001) also model the advantage of privately placed securities as reducing the information available to competitors.

industry increase their spending on market share acquisition. The innovator spends more because market share is now more valuable and the rival spends more to defend its position. Of course, both would prefer to spend less, but cannot credibly commit to doing so. A pooling equilibrium thus becomes both possible and attractive to *both* firms. In this case, the innovating firm secures private financing (which helps to hide its profitability and thus its new status) and then chooses the equilibrium spending of a less profitable firm (i.e., firm without the new technology). Here, private financing allows this technologically superior firm to remain within an equilibrium in which it spends less on acquiring market share. This in turn reduces the rival's market share spending leading to overall increased instantaneous earnings. Note it pays for the rival to adopt a "hear no evil, see no evil" strategy since finding out the truth would actually reduce its profits.

An important advantage of the model developed here is that it is well-suited for empirical analysis. The model can generate useful parameter estimates and predictions regarding variation in innovation and financing choice both within and across industries. A disadvantage is that its mathematical structure will not apply to all industries. That, perhaps, should not be too surprising as it seems unlikely that any one model can properly describe every industry there is. The paper highlights both the model's empirical uses and limits by first presenting estimates of the ease with which firms can acquire market share (ϕ). The industries for which this is accomplished is not exhaustive (in fact, given that the model describes an oligopoly, industries with too many firms are excluded before attempting to generate any estimates). Nevertheless, they span a fairly broad array of 55 oligopolies. These and other parameter estimates are then used in examples to highlight the model's quantitative predictions.

As an example of the model's ability to generate quantitative as well as qualitative predictions consider an innovation that increases the attractiveness of a firm's product by 50%. Based on the model's estimated parameters, if private financing delays a competitor's ability to copy the innovation by two years then this is worth $1.57 million to the average firm in the Carpet and Rug industry and $33.91 million to the average firm in the Dolls and Stuffed Toys industry. If the delay is 10 years the values increase to $6.31 million and $144.26 respectively. In principle these figures, and others like them, can be used to test the model in a valuation context by comparing them in the appropriate situations with the market's immediate reaction to the innovation's revelation and the subsequent performance of each competitor.

Within the existing literature the paper that comes closest to this one is Maksimovic and Pichler (2001). They examine the public-private financing decision within an industry that produces a homogenous good over two periods and for which there is costly entry and exit. In contrast, this paper examines a duopoly in a heterogeneous goods industry over an infinite horizon. Other contrasts between the papers are discussed later on in the text.

The paper is organized as follows. Section I presents the basic model. Section II examines the solution in the infinite horizon case. Section III looks at the case where an innovation improves a firm's ability to attract customers away from its rival and includes empirical analysis and model calibration. Section IV examines the case where the innovation increases a firm's profit per unit of market share. Section V examines the relationship between this paper and the prior literature. Section VI concludes. Finally, the Appendix contains details regarding the derivation of the model's equilibrium.

I. Model

A. Players, Timing, Dynamics and Strategies

The Lanchester (1916) battle model was originally designed to study military strategy. Since then variants have been widely used in the marketing literature to examine advertising strategies (see e.g., Erickson (1992); Erickson (1997); Fruchter and Kalish (1997); for a review, see Dockner, Jørgensen, Van Long, and Sorger (2000)).[6] Here it is adapted to produce a differential game within which to explore competition among duopolists faced with opportunities for new innovations and financing choices.

Consider two risk neutral value maximizing firms battling for market share. Call the firm that faces the innovation and financing decision "Firm 1." The rival is "Firm 2." Let $u_i(t)$ be the dollars spent by firm $i \in \{1,2\}$ on gaining market share at instant t. Let s_i denote the effectiveness of spending. Note that spending to acquire a competitor's customers (u_i) can imply a wide range of activities including advertising, new product design, store openings and R&D. The s_i parameters can represent the relative attractiveness of each firm's product and/or the relative quality of their marketing campaigns. Innovation opportunities in the spending effectiveness (s_i) parameter will be the primary focus in Section III.A.

The market share of Firm 1 at time *t* is denoted $m(t)$. Firm 2's market share is then $1 - m(t)$. Time is continuous and there is a finite starting point at $t = 0$. Given the initial condition $m(0)$, m evolves as follows:

$$dm = \frac{\phi\left[(1-m)s_1u_1 - ms_2u_2\right]}{s_1u_1 + s_2u_2}dt \tag{1}$$

[6] Although, to our knowledge, not in the form presented here.

where ϕ represents the speed with which consumers react to each firm's entreaties. Intuitively, (1) says that the variation in Firm 1's market share is simply the difference between what it gains from Firm 2's market share and what it loses to Firm 2.[7] The market share of Firm 1 increases with its own spending and effectiveness (u_1 and s_1, respectively) and decreases with spending and effectiveness of the competitor's spending.

Note that high current $m(t)$ gives Firm 1 "more to lose" to Firm 2 and as a result makes it easier for Firm 2 to gain market share.[8] Since this paper seeks to examine economic outcomes within industries that are natural oligopolies an assumption about consumer behavior like this is needed. If the industry is characterized by positive network externalities then it is a natural monopoly. In this case once a firm's market share reaches a tipping point it eventually acquires all of the market. But, in a natural oligopoly that cannot be the case. Instead, it must be that some consumers find each firm's product more attractive even if most use its rival's product. For example, McDonalds is the largest fast food restaurant chain in the U.S. Nevertheless, many people only eat at its rival Burger King. Thus equation (1) implies that this is a model where firms produce heterogeneous products and consumers have heterogeneous preferences.

Instantaneous profits are assumed to be proportional to market share. Let α_i denote the revenue generating ability of firm i per unit of market share. Profits π equal revenues minus both spending on market share competition and a fixed operating cost f_i:

$$\begin{aligned} \pi_1(t) &= \mathrm{e}^{gt}\,(\alpha_1 m(t) - u_1(t) - f_1\,) \\ \pi_2(t) &= \mathrm{e}^{gt}\,(\alpha_2(1-m(t)) - u_1(t) - f_2\,) \end{aligned} \tag{2}$$

[7] The model can be modified to include a stochastic dm. The results are unchanged since the firms are risk neutral profit maximizers. It is also straightforward to modify it to allow for J firms, and this is done in Section III.C in order to estimate the model parameters.

[8] In the marketing literature researchers tend to use as the law of motion either: (1) $dm/dt = u_1(1-m)-u_2m$ or (2) $dm/dt = u_1\sqrt{1-m} - u_2\sqrt{m}$ (Dockner et al. (2000)). One advantage of using (1) instead is that it is unit free. This eliminates the problem that changing the unit of currency also changes the rate at which m changes over time.

The term g represents the industry's rate of growth. It is assumed that as the industry grows larger profits and costs grow proportionately.[9] Firm 1 is currently financially constrained and has secured financing sufficient only to finance equilibrium spending in the current competitive environment.[10] In Section, IV, we examine a setting in which the innovation opportunity is in firms' revenue-generating ability (α_i).

To help streamline the exposition details regarding the derivation the model's equilibrium conditions can be found in the Appendix. There a general version is solved. Each of the following sections then employs that general solution to discuss the interactions between firms and their financial structure in various special cases. Thus, in the main body of the paper equilibrium conditions are simply stated without proof except for occasional references back to the Appendix.

B. The Equilibrium Value Functions

Let r denote the instantaneous discount rate. Assume $r>g$ and let $\delta=r-g$. Assume neither firm ever exits. Following standard practice in the literature on differential games the analysis seeks a Nash equilibrium in which the players use Markovian strategies (see Dockner, Jørgensen, Van Long, and Sorger (2000)). The Appendix shows that each firm's value function V_i at time t (i.e., the present discounted value of each firm's profit stream conditional on the equilibrium strategies) can be written as:

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[9] Equation (2) normalizes the industry starting value to one. In general for a starting value of π_0 multiply each equality by that amount. Since this has no impact on the model's equilibrium conditions the π_0 are suppressed to reduce the notational burden.

[10] Note, that spending by each firm does not impact the industry growth rate. Thus, the model should be thought of as applying to an industry in which innovations tend to change customer loyalties rather than increase overall demand. For example, an easier to swallow aspirin will probably cause consumers to switch brands but seems unlikely to lead to an overall increase in pill consumption. One can modify the model to allow g to depend on the u_i but at the cost of a closed form solution.

$$V_i(m,t) = a_i(t) + b_i(t)m \tag{3}$$

within the scenarios considered in this paper.[11] The terms a_i and b_i are functions of time and as shown in the Appendix equal:

$$a_1(t) = \delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - f_1\right] + C_1 e^{\delta t}, \tag{4}$$

$$a_2(t) = \delta^{-1}\left[\frac{\phi\alpha_2^3 s_2^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} + \frac{\delta\alpha_2}{\phi+\delta} - f_2\right] + C_2 e^{\delta t}, \tag{5}$$

$$b_1(t) = k_1 e^{-(\phi+\delta)(T-t)} + \alpha_1(\phi+\delta)^{-1}, \tag{6}$$

and

$$b_2(t) = k_2 e^{-(\phi+\delta)(T-t)} + \alpha_1(\phi+\delta)^{-1} \tag{7}$$

where the constants C_i and k_i depend upon a particular problem's boundary value conditions (i.e., the value of the V_i terms at some terminal date T).[12]

II. Full Information, Infinite Horizon Equilibrium

The simplest version of the model involves two firms that do not innovate, do not need outside financing, and compete over an infinite horizon. Since the game lasts forever the solutions to the model must be time independent. Thus, in equations (4) through (7) the C_i and k_i must all equal zero.

Since $\partial V_i / \partial m$ equals b_i one can now plug equations (6) and (7) (with k_i equal to zero) into (28) and (29) to find each firm's equilibrium spending on customer acquisition of:

[11] Extending the model to J firms yields value functions that are qualitatively similar to the two-firm case.

[12] There are, of course, boundary conditions under which the solutions given above will not hold. However, for the problems considered in this paper equations (4) through (7) fully characterize each firm's equilibrium value function.

$$u_i^* = \frac{\phi \alpha_i^2 \alpha_j s_1 s_2}{(\phi + \delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} \tag{8}$$

for *i* equal to 1 or 2 and $j \neq i$ (see the Appendix for their derivation). The optimal control (u_i^*) is decreasing in the discount rate net of growth ($\delta = r - g$); increasing in own-firm revenue generating ability (α_i), and increasing in the speed with which consumers react to their attempts to gain market share (ϕ). Optimal spending (u_i^*) is also dependent on competing firm revenue-generating ability and on both firms' spending efficiency (α_j, s_j and s_i, respectively). These comparative statics naturally depend on the α and s of each firm. Intuitively, in competitive markets, firms may be forced by competitive pressures to spend aggressively to attract customers when their rivals also have incentives to do so; however, if their rivals do not have such incentives (if they are not very profitable or if they are not effective at attracting customers), profit-maximizing firms will limit spending. Using available accounting data these conclusions can be tested empirically. Also, note that the marginal value of market share for a given firm (b_i) depends only on its own firm revenue generation ability, α_i, which is yet another testable result.

To gain further insight into the impact of the model's parameters on equilibrium behavior consider what happens in the steady state: $dm = 0$. From (1) if dm equals zero, then the steady state market share m^* equals,

$$m^* = \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}. \tag{9}$$

Clearly, both increased revenue generating ability and the efficiency of spending increases a firm's equilibrium market share. Overall industry concentration (sum of squared market shares) increases in $|\alpha_1 s_1 - \alpha_2 s_2|$. Thus one can think of the product of $\alpha_i s_i$ as a firm's competitive ability and the difference $\alpha_i s_i - \alpha_j s_j$ as i's competitive advantage relative to its rival. Note, that ϕ drops out of (9). In the long run it is irrelevant how long consumers take to react to each firm's attempts to acquire market share so long as they react at all.

Setting the C_i and k_i to zero in (4) through (7) and plugging the resulting values into (3) allows one to write out the explicit solutions for the value functions in the current case. An examination of the results yields the counter intuitive conclusion that if m is at its steady state value then decreasing consumer responsiveness (ϕ) increases the value of both firms.[13] The reason for this can be found in the equilibrium values of u_i and the fact that the steady state value of m does not depend on ϕ. (For the latter see equation (9).) From (8) both firms will reduce their spending on market share competition if consumers become less reactive. Thus, both firms benefit from ϕ's reduction since they then earn the same steady state revenue stream while wasting fewer resources trying to lure away each other's customers. Formally then, $\partial V_i / \partial \phi \big|_{m=m^*} < 0$ for both firms. Effects of this type are easily seen in real industries. For example, if beer drinkers exhibited greater loyalty to particular brands brewers would undoubtedly advertise less, and collectively earn higher profits. From 1981 to 2004 per capita beer consumption in the U.S. fell from 24.6 to 21.6 gallons despite heavy product advertising (USDA, 2005). But, no one brewer can reduce its own spending without losing customers to competitors. Thus, in equilibrium, they end up advertising just to retain their current market share even amid stagnant sales. Compare this to the situation in, for example, natural gas

[13] Again, one can test this using available accounting data.

distribution where consumers are locked into a single supplier and thus these firms do relatively little advertising.

A distinguishing feature of any industry is the ease with which producers can steal away each other customers, represented by the value φ in the model. However, additional economic intuition can be gained by looking at what can be called the industry "half-life." This represents the time it would take a firm to lose half its customers if it stopped working to keep them by setting u_i to zero. This value can be calculated from (1) and turns out to equal $h=\ln(2)/\varphi$.

While the half life (h) is the time that it would take a firm to lose half of its market share if it stopped spending to attract customers it can also be used to analyze the growth of new firms. If a firm enters an industry its rival will not passively let it grow. This naturally slows the entrant's growth making it impossible to capture half the market in the time interval h described above. How long would it take? The model can be used to provide a quantitative answer. Imagine an industry dominated by a single firm when a rival enters with an equally attractive product ($s_1=s_2$) and profit per unit of market share ($\alpha_1=\alpha_2$). In this case $u_1^* = u_2^* = \phi\alpha / (4(\phi+\delta))$. Plugging these values into (1) implies $dm = \phi(1-2m)dt$. As expected the entrant stops growing when it has half the market. Integrating the resulting ODE and setting $m(0)=0$ generates a market share for the rival over time of $m(t) = .5 - .5e^{-2\phi t}$. Thus the new entrant firm can be expected to capture a quarter of the market after $t=h$ years. In this example, the incumbent's reaction to the entrant cuts the entrant's rate of growth in half.

Growing, newly public firms can be difficult to value, in part because of challenges associated with forecasting their future cash flows. Suppose firms, in the above example, go public upon reaching a market share of .25. Again solving the ODE one finds that the entrant's growth decelerates from its pre-IPO levels. While the firm took over a quarter of the market in the first h years of its life it will now take the same amount of time to go from a quarter to three-eighths. What this means, however, is that if one can estimate the value of h associated with a particular industry and each firm's s and α then the model not only makes predictions about each firm's long run market share, profitability, spending on customer acquisition but also regarding the time it takes new entrants to reach particular market share levels. These estimates should also help predict cash flows and improve valuation.

III. Financing and Innovation

Within the model firms can improve their profitability by adopting innovations that increase their value of s and α. But changes in each offer potentially very different strategic options. The s_i represent the effectiveness of each firm's spending to garner market share. In real economic terms, an increase in s_i should thus correspond to an increase in the attractiveness of i's product. This can occur either through an innovation (such as a washing machine that cleans faster) or an improved advertising campaign (Nike's use of Michael Jordan as a spokesman). Whatever the case, it is clear that unless consumers are aware of the innovation or advertising campaign then s_i cannot change. If changes to s_i can only occur if consumers know of it then it must also be the case that rival firms must know as well. Equivalently, the current value of each firm's s is always common knowledge. Now consider α_i which represents a firm's profits per unit of market share. Unlike s there is no reason α must be publicly known at all times, at least until some sort of financial report (like a 10-Q) is released. This means that changes in s and α can lead to very different equilibrium dynamics both because they influence profits in unique ways and because they have different implications regarding the equilibrium information sets. These issues are explored in greater detail below.

A. Innovations in s

The innovation game analyzed here assumes that at time 0 Firm 1 observes an opportunity to improve its spending effectiveness from s_1 to s_1^* at time T_1. To develop the technology Firm 1 must first incur a cost of *Z*. Firm 1 is currently private. To make the public/private financing question interesting, assume it only has enough capital to sustain equilibrium spending in the current competitive environment. If Firm 1 decides to innovate, it must secure financing of *Z* for the project.

At time zero Firm 1 must choose among the following strategies:[14] (i) innovate and finance publicly, (ii) innovate and finance privately, or (iii) do not innovate. If Firm 1 decides not to innovate spending effectiveness remains at S=$\{s_1, s_2\}$ forever. Firm values are then given by the solutions derived in Section II. Adopting the technology means that, after a period of development from T_0 to T_1, Firm 1 enjoys a first mover advantage in the product market. This competitive advantage does not last forever; Firm 2 eventually copies the technology and increases its spending effectiveness to $s_2^* > s_2$. The paper assumes that the relative spending effectiveness remains constant whenever both firms employ the same underlying technology. That is, $s_1 / s_2 = s_1^* / s_2^*$. Under this assumption, competition eventually drives each firm's profitability back to pre new technology levels. However, markets experience real efficiency improvements via the adoption of the innovation.[15]

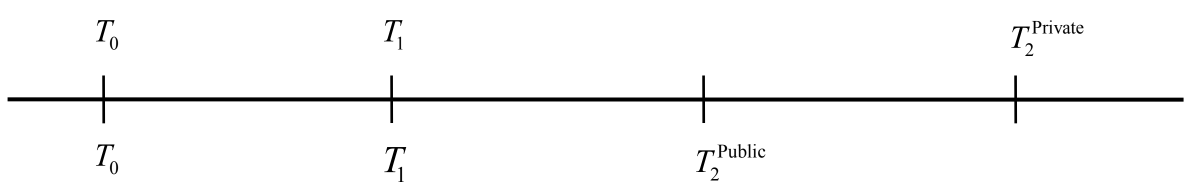
[14] We are assuming that the opportunity to innovate expires immediately if it is not taken. This might be expected in actual product markets. For example, when technologies can be patented, a firm's decision to abandon a potential development may open opportunities for other firms to profit from development. For a model that assumes that the option to go public at a later date is valuable, see e.g., Benninga, Helmantel and Sarig (2005).

[15] It is easy to relax this assumption. However, we believe that $s_1 / s_2 = s_1^* / s_2^*$ is more realistic since it captures the idea that, in competitive markets, innovations are eventually adopted by all firms in an industry.

The public versus private financing decision has important value implications. Public financing is cheaper than private financing (i.e., there is a private market discount). This is due to the smaller pool of investors in private markets as well as the relative illiquidity of these markets. On the other hand, if Firm 1 decides to finance the project via a public offering, regulatory disclosure requirements force him to reveal investment behavior and financial data that increases the speed at which Firm 2 is able to successfully copy and adopt the technology.[16]

To capture the above conditions assume that if Firm 1 is privately financed Firm 2 will not be able to duplicate the technology until date T_2^{Private}. Conversely, if Firm 1 instead finances publicly then the required disclosures allow Firm 2 to begin the process of copying the technology somewhat earlier. In this case, Firm 2 successfully adopts the innovation by T_2^{Public} where the relative dates satisfy $T_2^{\text{Private}} > T_2^{\text{Pulic}} > T_1$. Upon Firm 2's successful adoption of the new technology s_2 increase to s_2^*.

Figure 1: Innovation and Financing: Timeline



T_0: T=0 Firm 1 decides whether to adopt innovation s_1^*. If innovation occurs, he decides how to finance it (publicly or privately). Development stage begins and S={s_1, s_2}. If no innovation, firms play full information, infinite horizon differential game with S={s_1, s_2}.

T_1: Firm 1's innovation becomes effective. S={s_1^*, s_2}.

T_2: Firm 2 successfully copies and adopts the innovation. S={s_1^*, s_2^*}.

[16] We assume that copying is costless. It might be more realistic to assume that the cost of copying is positive (though much less than Z). As long as the copying cost is sufficiently small that Firm 2 finds it worthwhile to copy, results and intuition regarding Firm 1's innovation and financing decision are identical to the costless copying case.

Consider the IPO of Coley Pharmaceutical, a biotechnology firm engaged in cancer therapy research. According to Lähteenmäk and Lawrence (2006), this was the biotech sector's largest U.S. IPO in 2005. While innovation opportunities abound in the biotechnology industry, the Coley case fits particularly well with the model's assumption that the innovation primarily draws customers from rivals rather than speeds up industry growth.[17] At time zero, Coley observes the opportunity for a breakthrough in cancer immune therapy; if financed, it takes until T_1 to bring the technology to market. If publicly financed, rivals can observe Coley's investment patterns and perhaps glean other information from its mandated disclosures and "road show" documents to begin early development of the technology, leading to the erosion of the length of Coley's competitive advantage.[18] In real life, Coley initially financed the development privately and waited until T_1 (market stage when there was no longer any information to hide) to undertake the public offering.[19] The goal in the following section is to provide estimates of the value of the initial decision to remain private to a firm like Coley.[20]

1. Value and Incentives to Innovate: Public Financing

To determine Firm 1's optimal decision rule one needs to compare its value under each of the three possible scenarios: (1) do not innovate; (2) innovate, finance publicly; and (3) innovate,

[17] People with cancer may switch to Coley's treatment; however, it seems highly improbable that consumers will now seek to increase their consumption of cancer related drugs

[18] While patents help they do not block competitors from developing competing products. Once a drug shows promise rival firms see if they have drugs in their pharmacies that have similar effects and that might therefore generate similar treatment benefits. It is not a coincidence that there are now over a dozen cholesterol reducing statins on the market.

[19] Of the Top 10 IPOs of biotechnology firms of 2005, Lähteenmäk and Lawrence (2006) report that: 1 had a product at the market stage; 5 had products in Phase 3 development; and 4 firms had products in Phase 2. Importantly, none had products in Phase I development, pre-clinical testing or discovery. An important observation is that the industry has two large incumbents: Amgen and Genetech. Our model provides the testable implication that the relative dominance of these two firms plays an important role in the going public decisions made by others in the industry.

[20] The competitive benefits of private financing are not unique to the biotechnology sector. For example, see Google's S-1 filing (for its 2004 IPO) and the quote discussed in the introduction.

finance privately. As usual, the solutions to each game are obtained by working backwards from when both firms successfully adopt the innovation. Since the intermediate steps provide limited economic insight they are relegated to the Appendix. After working through the algebraic details the value functions at t=0 under public financing are:

$$\begin{aligned}V_1^{Public}(T_0) &= \frac{\alpha_1}{\delta(\phi+\delta)} \frac{\phi\alpha_1^2 s_1^2}{(\alpha_1 s_1+\alpha_2 s_2)^2} \\ &+e^{-\delta T_1} \frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}-\frac{\alpha_1^2 s_1^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right] \\ &+e^{-\delta T_2^{Public}} \frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}-\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}\right]-\frac{f_1}{\delta}+\frac{\alpha_1}{\phi+\delta}m(0)\end{aligned} \tag{10}$$

and

$$\begin{aligned}V_2^{Public}(T_0) &= \frac{\alpha_2}{\delta(\phi+\delta)} \frac{\phi\alpha_2^2 s_2^2}{(\alpha_1 s_1+\alpha_2 s_2)^2} \\ &+e^{-\delta T_1} \frac{\phi\alpha_2}{\delta(\phi+\delta)}\left[\frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}-\frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right] \\ &+e^{-\delta T_2^{Public}} \frac{\phi\alpha_2}{\delta(\phi+\delta)}\left[\frac{\alpha_2^2 s_2^{*2}}{(\alpha_1 s_1^*+\alpha_2 s_2^*)^2}-\frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^*+\alpha_2 s_2)^2}\right]-\frac{f_2}{\delta}+\frac{\alpha_2}{\phi+\delta}\left(1-m(0)\right)\end{aligned} \tag{11}$$

for Firms 1 and 2 respectively.

While equations (10) and (11) look rather daunting they are in fact quite simple. The first term on the right represents the present value of each firm's profits until Firm 1's new technology comes on line. The second term equals the present value of each firm's profits during the period when Firm 1 has a technological advantage. The third term represents the final time period when Firm 2 catches up technologically. The final set of terms adjust the value function for the firm's fixed operating costs and current market share. Intuitively, one can think of the expression $\alpha_i s_i/(\alpha_1 s_1+\alpha_2 s_2)$ as representing a firm's relative competitive strength ($\alpha_i s_i$)

exclusive of the impact of its current market share. As time progresses, the s_i terms change in each fraction to track each firm's current technological level.

The observation that competitive environments greatly reduce the potential value of technological improvements routinely appears in the statements made by high-tech company executives. Their common complaint is that development of Microsoft compatible software is hindered by competition with Microsoft itself. The scenario typically outlined is that of a firm which produces an innovation and succeeds in acquiring customers. If this happens the claim is that Microsoft then copies the innovation (by building it into its existing products) and thereby steals away the innovator's market share. For example, a 2001 article on CNET News.com starts with,

> The battle over today's instant messenger market is vintage Microsoft, whose strategy enemies call "the three E's" in a parody of the company's marketing mantra: Embrace a rival's technology, extend it to work best with Windows, and extinguish the competition.
> Hu (2001)

The value functions (10) and (11) capture these kinds of industry dynamics and allow one to estimate the value of innovating given Microsoft's competitive response. Compared to a single firm setting, where T_2 is effectively set to infinity, the above scenario, in which Microsoft can discover a way to incorporate the innovation into its own product line at date T_2 can greatly reduce, if not eliminate, an innovation's value to its discoverer.

2. Comparative Statics: Incentives to Innovate

Before examining the question of how an innovation should be financed the first question to be addressed is how competitive forces impact the decision to innovate at all. This section thus examines how the incentives to innovate vary with the size of the innovation and with the size/profitability of Firm 1 and Firm 2.

The main results presented so far demonstrate how competitive pressures in real markets can significantly reduce the value of innovations that might appear worthwhile if considered in isolation. The comparative statics analysis allows us to shed additional light on the issue of where (within industries) innovations can be expected to occur. To make it clear whether or not Firm 1 has a competitive advantage within each equation let $\psi = s_1 / s_2 = s_1^* / s_2^*$ and $\psi^* = s_1^* / s_2$.

Then one can rewrite the value function of Firm 1 as:

$$
\begin{aligned}
&V_1(T_0) = \frac{\phi\alpha_1^3\psi^2}{\delta(\phi+\delta)(\alpha_1\psi + \alpha_2)^2} + \\
&e^{-\delta T_1}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2\psi_2^{*2}}{(\alpha_1\psi^* + \alpha_2)^2} - \frac{\alpha_1^2\psi^2}{(\alpha_1\psi + \alpha_2)^2}\right] + \\
&e^{-\delta T_2}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2\psi^2}{(\alpha_1\psi + \alpha_2)^2} - \frac{\alpha_1^2\psi^{*2}}{(\alpha_1\psi^* + \alpha_2)^2}\right] - \frac{f_1}{\delta} + \frac{\alpha_1}{\phi+\delta}m(0).
\end{aligned} \tag{12}
$$

The derivative of Firm 1's value function with respect to the magnitude of the competitive advantage during period T_1 to T_2 is thus:

$$
\frac{\partial V_1(0)}{\partial \psi^*} = \left[\frac{\psi^*}{(\alpha_1\psi^* + \alpha_2)^2} - \frac{\alpha_1\psi^{*2}}{(\alpha_1\psi^* + \alpha_2)^3}\right]\left[\frac{2\phi\alpha_1^3(e^{-\delta T_1} - e^{-\delta T_2})}{\delta(\phi+\delta)}\right]. \tag{13}
$$

This is clearly positive (as would be expected). What is of greater interest is the question of how the incentive to innovate varies with Firm 1's characteristics, in particular, Firm 1's revenue generating ability (α_1):

$$
\begin{aligned}
\frac{\partial^2 V_1(0)}{\partial \psi^* \partial \alpha_1} = &\left[\frac{6\alpha_1^2\psi^{*2}}{(\alpha_1\psi^* + \alpha_2)^2} - \frac{4\alpha_1^3\psi^{*2}}{(\alpha_1\psi^* + \alpha_2)^3} - \frac{8\alpha_1^3\psi^{*2}}{(\alpha_1\psi^* + \alpha_2)^3} + \frac{6\alpha_1^4\psi^{*3}}{(\alpha_1\psi^* + \alpha_2)^4}\right] \times \\
&\left[(\delta(\phi+\delta))^{-1}\phi\alpha_1^3(e^{-\delta T_1} - e^{-\delta T_2})\right].
\end{aligned} \tag{14}
$$

This is always greater than zero and implies that larger, more profitable firms (recall that the equilibrium size of Firm 1 is $m^* = \alpha_1 s_1 / (\alpha_1 s_1 + \alpha_2 s_2)$), are more likely to adopt new technologies.[21] Basically, larger firms are already superior competitors: that is why they are large to begin with. The increase in s thus allows them to draw away a substantial number of additional customers. Then, even after they lose their competitive advantage at date T_2 it still takes the rival some time to reclaim its long run market share.

Now consider the incentive to innovate as a function of the rival firm's characteristics:

$$\frac{\partial^2 V_1(0)}{\partial \psi^* \partial \alpha_2} = \left[\frac{-4\psi^*}{(\alpha_1\psi^* + \alpha_2)^3} + \frac{6\alpha_1\psi^{*2}}{(\alpha_1\psi^* + \alpha_2)^4} \right] \left[\frac{\phi\alpha_1^3(e^{-\delta T_1} - e^{-\delta T_2})}{\delta(\phi+\delta)} \right]. \tag{15}$$

Equation (15) is greater than zero when $\alpha_1\psi^* - 2\alpha_2 > 0 \quad \Rightarrow \quad \alpha_1 s_1^* - 2\alpha_2 s_2 > 0$. Therefore, for small firms, an increase in the competitors' profitability and size makes the innovation less valuable. This is because an increase in Firm 2's α or s increases its marginal value of market share and makes Firm 2 more aggressive. This can impose a cost that is greater than the potential gains from adopting the technology. A similar exercise shows that increasing consumer responsiveness increases the value of any new innovation; differentiating (13) with respect to ϕ yields $\partial^2 V_1(0)/\partial\psi^*\partial\phi > 0$. Also note that an increase in ϕ is more valuable to smaller rivals. Translating this to the available data, ϕ should play a stronger role for firms small enough to make good use of venture capital financing (which can be limited in size) than for firms big

[21] The relationship between firm size and innovative activity has long been the subject of academic debate. See Kamien and Schwartz (1975) for a survey of early work and a discussion of the Schumpterian hypothesis that product market rivalry will impact innovation incentives. Acs and Audretsch (1987) find that large firms have the innovative advantage in industries that are capital-intensive with high concentration and advertising expenditure. They conclude that large firms have an advantage in imperfectly competitive industries, while small firms have a greater advantage in perfectly competitive industries. Blundell, et al. (1999) also find firms with larger shares engage in more innovative activity and that the market reacts by viewing their innovations as being more valuable. More recently, Hall (2005) traces some of the basic ideas investigated here to Schumpeter: that innovations can be copied by rival firms thereby decreasing incentives to invest. We not only explicitly model this possibility, but do so in a dynamic continuous time model.

enough to consider going public. For succinctness, Table 1 summarizes the comparative static results given above and a few others as well.

Maksimovic and Pichler (2001) examine closely related issues. They consider financing choice in a growing industry and the potential for herding by potential entrants. The cost of disclosure is letting rivals know how lucrative the business is. Their main result is that private financing occurs when start-up costs are high and when there is a high probability of displacement by a superior rival; public financing occurs when the technology is not costly and when the probability of displacement is low. Our focus differs. We study financing choice in a model with dynamic competitive interactions and a finite interval over which a competitive advantage can be maintained. We also explicitly examine the potential value implications of the existence of a strong rival for a growing firm in an industry with heterogeneous goods.

An advantage of the model developed here is that it can easily be fit to data that are readily available. Besides offering a potentially rich set of cross sectional predictions for future testing it also allows for the investigation of competitive dynamics along the equilibrium path. This too is well suited for real data. Because competition evolves through time one can use the model to make better use of the available panel datasets chronicling stock returns and corporate accounting statements. One possible mapping between the model's parameter values and variables available on CRSP and COMPUSTAT can be found in Table 2.

B. Private versus Public Financing

We now turn to the question of how potential innovators will finance their investment. In this case the value functions at time zero are analogous to those under public financing and given by equations (10) and (11). The only difference is that the T_2^{Public} terms are replaced by T_2^{Private}.

Label the value functions with this change V_i^{Private}. While the forms of the value functions are identical to the public financing case keep in mind that $T_2^{\text{Private}} > T_2^{\text{Public}}$ and thus Firm 1 enjoys first mover advantages for a longer time under private financing.

In concurrence with the empirical literature the model assumes that private financing is more costly than public financing (see e.g., Hertzel and Smith (1993) for evidence of this in the context of private placements). The simplest way to capture the increased cost of private financing is to build it directly into the technology's implementation cost. Thus, if the new technology costs *Z* to implement under public financing assume that its cost increases to *Z*(1+*D*) under private financing. Here, *D* is the private market discount. Clearly, when the private market discount (D) or the scale of the project (Z) are small, private financing is more likely. Assuming the benefits from adopting the innovation exceed its costs then some algebra shows private financing is preferred when:

$$\left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right] \frac{\phi \alpha_1^3}{\delta(\phi+\delta)} \left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2}\right] - ZD > 0. \quad (16)$$

Based on (16) it is now possible to produce a number of quantitative as well as qualitative predictions regarding when firms will choose one type of financing or another. Quantitative values will be discussed in Section C.2

1. Comparative Statics: Private versus Public Financing

This section examines how the incentives to finance publicly vary with firm characteristics. Recall from Section A.2 above that $\psi = s_1 / s_2 = s_1^* / s_2^*$ and $\psi^* = s_1^* / s_2$. With this, rewrite the value of private financing as:

$$V_1^{\text{Private}} - V_1^{\text{Public}} = \left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right] \frac{\phi \alpha_1^3}{\delta(\phi+\delta)} \left[\frac{\psi^{*2}}{(\alpha_1 \psi^* + \alpha_2)^2} - \frac{\psi^2}{(\alpha_1 \psi + \alpha_2)^2}\right] - ZD \quad (17)$$

We are interested in the cross-sectional relationships among the available financing choices and firm characteristics. First consider Firm 1's revenue generating ability, α_1. To simplify the notation, let $q = \left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right](\delta(\phi+\delta))^{-1}\phi$. Then:

$$\frac{\partial(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_1} = q\alpha_1^2\left[\frac{\psi^{*2}(\alpha_1\psi^* + 3\alpha_2)}{(\alpha_1\psi^* + \alpha_2)^3} - \frac{\psi^2(\alpha_1\psi + 3\alpha_2)}{(\alpha_1\psi + \alpha_2)^3}\right]. \qquad (18)$$

As shown in the Appendix equation (18) is positive for all $\psi^* > \psi$. Therefore, the incentive to secure private financing is increasing in Firm 1's revenue generating ability (α_1). Intuitively, this occurs because high α firms are in the best position to use any technological advance to aggressively pursue market share. Equation (18)'s prediction that the most lucrative projects should be financed privately is consistent with findings of operational underperformance of IPO firms in the years following issuance (e.g., Loughran and Ritter (1995)).[22] That is, initially firms finance their very high ψ^* projects privately and then go to the public markets when only more modest ψ^* innovations remain.

Now consider the private financing choice as a function of the rival's revenue generating ability (α_2):

$$\frac{\partial(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_2} = 2q\alpha_1^3\left[\frac{\psi^2}{(\alpha_1\psi + \alpha_2)^3} - \frac{\psi^{*2}}{(\alpha_1\psi^* + \alpha_2)^3}\right]. \qquad (19)$$

The sign of equation (19) depends on the relative values of α_1 and α_2. Incentives to remain private are decreasing in α_2 when $\alpha_2 > \alpha_1\psi/2$, and increasing in α_2 when α_2 is small (i.e., $\alpha_2 < \alpha_1\psi/2$). This is because profitable rivals (high α_2) will spend more aggressively during the period of Firm 1's first mover advantage, making it difficult for Firm 1 to capture market share. On the

[22] Chemmanur, He and Nandy (2007) report that total factor productivity of IPO firms increases prior to the IPO and then decreases in the years following the IPO. This peak in productivity at the time of the IPO is consistent with the prediction that the most lucrative projects are financed privately.

other hand, when α_2 is small, the benefits from extending the period of first-mover advantage outweigh the costs of higher equilibrium spending.

There are several additional observations. First, Firm 1's incentives to finance privately increases in the technological advantage the innovation provides (i.e., in ψ^*). Second, as one might then expect, the opposite is true of ψ. An increase in Firm 1's initial (and final) spending effectiveness relative to Firm 2 decreases Firm 1's incentive to issue private securities. From equation (17), observe that the incentives to remain private decrease when real interest rates are high. This provides an explanation for expected IPO patterns based on economic fundamentals that is distinct from the market timing arguments in Baker and Wurgler (2000).

Third, the incentive to remain private increases as the time gained from delaying Firm 2's adoption increases (this can also be interpreted as an increased incentive to avoid disclosure requirements).[23] That is: $\partial(V_1^{\text{Private}} - V_1^{\text{Public}}) / \partial T_2^{\text{Private}} > 0$ and $\partial(V_1^{\text{Private}} - V_1^{\text{Public}}) / \partial T_2^{\text{Public}} < 0$. This has implications for patent policy. Clearly, innovation can be good for consumers. Extending the incentive to innovate and finance publicly (via extending patent life and thus increasing T_2^{Public}) encourages innovation and eliminates the costs of private financing. Furthermore, because the model's parameters have real world analogs it can potentially help quantify to what degree an increase in patent protection will encourage the use of lower cost public financing.

Fourth, increasing consumer responsiveness (ϕ) increases the incentive to finance privately. The empirical section that follows offers some support to for this prediction. Fifth, the private market discount decreases Firm 1's incentives to remain private as it clearly makes

[23] An obvious implication is that, if the private market discount is sufficiently high (making private financing prohibitive), disclosure requirements can inhibit innovation.

such financing more costly. One can find a summary of the section's results as well as several additional comparative statics in Table 3.

C. Estimation

One important advantage of the main model is that it is well-suited for empirical analysis. Equation (1) provides a mechanism through which the consumer responsiveness parameter ϕ and spending effectiveness parameters s_i can be estimated for each industry. Because accounting data arrive in discrete time intervals the equation must be adapted. Also, real industries generally have more than two firms so the equation is extended to accommodate this as well. In this case, the law of motion for market share for firm i in an industry with J competitors becomes:

$$m_{i,t+1} - m_{i,t} = \frac{\phi u_{i,t+1} s_i}{\sum_{j=1}^{J} u_{j,t+1} s_j} - \phi m_{i,t} dt. \tag{20}$$

Since the market shares have to add to one, there are only J-1 independent observations in each period. Thus, the Jth observation in each period is dropped. Next, note that dm is homogenous of degree 0 in the s_i terms. For identification, we restrict the sum of s_i to equal 1. Given the $T(J-1)$ independent changes in market shares and further assuming that ϕ remains constant over the sample period, one can estimate the parameters in Equation (20) by minimizing the total sum of squared errors via non-linear least squares.[24]

[24] In reality, firms enter and exit industries over time, so there are $T(J_t$-1) observations for each industry.

1. Data and Estimation

Data and Sample Selection

To estimate ϕ, it is necessary to obtain data on both market shares and spending by all firms in the industry. Market share, $m_{i,t}$, is defined as share of sales of all *CRSP/COMPUSTAT* firms in the *COMPUSTAT* 4-digit SIC code. While broader industry categories are often used in the finance literature (e.g., 2-digit and 3-digit SIC codes), we choose to use the finer 4-digit codes in an effort to capture industry dynamics in the best possible way.[25] Spending, u_i, is defined as the sum of: capital expenditures, research and development and advertising. The analysis covers a broad cross-section of industries and the spending proxy is chosen to be sufficiently general to capture market share competition in a wide range of competitive structures.[26]

The initial sample consists of all *COMPUSTAT* 4-digit SIC codes for which there is non-missing information on sales and spending in *COMPUSTAT* and in which there are fewer than fifty firms. The oligopolistic structure described in the main model makes industries with large number of firms inappropriate in the context of this paper.[27] We begin the estimation period in 1990 and use all available data through 2005. While data are available for a much longer time

[25] *COMPUSTAT* codes are used due to findings in the literature that linkages among firms (e.g., return correlations) are higher than with CRSP SIC codes. For example, see Guenther and Rosman (1994).

[26] Advertising has been used to estimate variants of Lanchester models the marketing literature. See e.g., the empirical specification in Chintagunta and Vilcassim (1992).

[27] We exclude financial services sectors (SIC codes 6000-6999) and conglomerates (code 9997). We also exclude all industries with less than two publicly traded firms during the entire sample period (1990-2005). The main model assumes no exit; however, due to changing product mix and SIC code re-classification for some firms, we adjust for "entry" and "exit" in the data by assuming that each industry firm loses and gains market from the entrant and exiting firm, respectively, in proportion to their current market share. That is, let m_C_i equal each firm's market share calculated from the firms in the industry listed on COMPUSTAT at date *t*. Using this notation, the estimates assume that: $m_i(t) = m\_C_i(t)\left[\frac{Exiting\ Firm\ Share(t)}{1-Exiting\ Firm\ Share(t)} - New\ Firm\ Share(t+1)\right]$ for each industry..

horizon, we focused on the years 1990-2005 based on the idea that market definitions (and therefore estimation parameters) might change over very long periods.[28]

ϕ and Industry Half Life Estimates

Table 4 reports estimated industry ϕ's, their standard errors, and the half lives (h) based on the point estimates for ϕ. There are fifty-five SIC codes meeting the data filtering requirements described above in which estimates for consumer responsiveness (ϕ) and spending effectiveness (s_i) were obtained for all firms.[29] Individual s_i are estimated for all 499 firms within these industries; however, they are not reported in the table (for brevity).

The values in Table 4 range from 1.03 years in the "Forestry" industry to a (likely overestimated) value of 9.76 years in the "Petroleum Bulk Stations" industry. The median four digit industry has a half life of 2.63 years and the interquartile range is 1.76 to 3.86 years. Standard errors are presented in the table; however, cell by cell t-statistics are not reported since it is unclear what the null hypothesis should be. Furthermore, the primary goal is simply to provide some economic intuition as to what parameter estimates imply.

Economically, the question is whether the half lives in Table 4 are "reasonable." Recall, that setting u_i to zero does not imply that the firm ceases operations, maintenance activities, or eliminates all customer service. Rather, it means that it does not actively compete for customers through things like advertising, R&D, and the construction of new outlets. In this light the

[28] Although we also estimated the model using data from years 1972-2005, we focus on results from a shorter estimation period (years 1990-2005). An alternative approach to possible changing industry structure over time would be to allow for time-varying φ and s.

[29] Weakening the latter restriction (estimates for all s_i) by imposing the restriction that $s_i \equiv s_j \equiv 1$ would make the industry sample size increase nearly six-fold. We acknowledge that the model does not describe the structure of all industries, so we prefer to focus on industries in which we are able to obtain estimates for all parameters of interest.

estimates seem plausible. For example, the estimated half life for the amusement park industry is 1.6 years, whereas the half life for the dolls and stuffed toys industry is more than double that number, at 3.4 years. Here, the intuition is that improvements such as the introduction of new rides may cause families to choose an innovator's park for their next amusement park outing, whereas the goal of building a child's supply of toys with a particular character or theme may make them more reluctant to buy a competitor's product (for example, consider Mattel's extensive *Barbie* collection of dolls, toy vehicles and accessories). This variation in consumer loyalties would seem to make these reasonable estimates. While there are clearly some industries with estimated half lives that appear to be either too high or low most seem within the range one expects.

2. Calibrations

In this stage of the analysis, we calibrate the model. The main focus is on the economic value of remaining private. While not reported in Table 4, s_i was estimated for every firm in addition to the industry φ. In addition, we estimate parameters α_i and fixed cost f_i based on equation (2), $\pi_i(t) = e^{gt}(\alpha_i m_i(t) - u_i(t) - f_i)$. Where $\pi_i(t)$, $m_i(t)$ and $u_i(t)$ are earnings, market share and spending of firm *i* during year *t*, respectively. The e^{gt} term is assumed to be the ratio of total current period industry sales to total industry sales (using real 2005 dollars) in the first year in which we observe the industry in the data. Given these parameters and equation (17), we calculate $\Delta V = V_{private} - V_{public}$ under the assumptions that: (1) the firm invests in an innovation that increases its *s* by 50%; (2) industry growth is 2% and; (3) T_1=2 and T_2^{Public} = 5. Real discount rates are calculated using equity betas from a market model estimated over the 60 months preceding year t (unlevered betas are calculated using book values of debt and assuming β_D=0).

We obtain estimates for all industries in the sample; however we present four sample industries in Table 5 for illustrative purposes: Carpets and Rugs (SIC 2273); Plastics and Foam Products (SIC 3086); Dolls And Stuffed Toys (3942); and Detect,Guard and Armor Car Services (7381). These industries reflect significant between-industry variation in consumer responsiveness (ϕ of .13 to .49, respectively), as well as within-industry variation in profitability. The calculations in Table 5 set ZD=0; however, it is easy to apply a private market discount. Given the evidence in Hertzel and Smith (1993), 20% would be an appropriate approximation. This implies that for a required investment of $20 million, firms in Carpets and Rugs industry would typically not find it profitable do so if it extends competitive advantage by only two years (*ZD* = $4M>1.57M).[30] If private financing extends private information in that industry by a full ten years, average Carpet and Rugs firms would take the investment (*ZD* = $4M<6.31M). Detect, Guard, Armor Car Services firms need a full 15 extra years of competitive advantage in order to find it profitable to opt for private financing. On the extremes, Plastics and Foam Products firms would almost never take the investment and private financing, while Doll and Toy industry firms would almost always opt for private financing of this $20 million investment. Observe that estimated α's are higher in Dolls and Toys industry than in the others. This is consistent with the idea that profitable firms have incentives to remain private.

Table 6 shows industry averages; however, there is also substantial within-industry variation in the value of financing innovations in s privately. Figures 2(a)-2(d) show individual firm values from obtaining private rather than public financing, where the time horizon over

[30] $4M stands for 4 million U.S. dollars.

which private financing extends the period of first mover advantage varies from 1 to 15 years (i.e., T_2 from 6 to 20) for the four sample industries. From the figures, it is easy to see that current spending effectiveness, market shares and profitability and all have important impacts on firms' incentives to remain private. Assuming a 20% private market discount, the calibrations suggest that a firm in the Carpet and Rug industry like Mohawk (Figure 2a) would be willing to privately finance an investment in s that costs less than \$3.77M, even if it extended the first-mover advantage period by only one year. A private firm similar to its competitor, Dixie, would only be willing to finance the investment privately if it cost less \$137,000. Within the Doll and Toy industry (Figure 2c), the estimated variation in α and s implies that firms would be willing to finance investments of up \$6.48M (Mattel) to as high as \$29.01M (Boyds Collection) if they extended the period of first mover advantage by a year.

The analysis presented in this section is intended to provide examples of how the model in this paper can be applied in empirical research. Specific industry analysis using richer datasets than the annual COMPUSTAT tapes (e.g., detailed brand-level advertising and sales information; higher frequency data; international industries and firms) would shed even more light on the impact of industry dynamics on the innovation and financing decisions of firms. Moreover, the use of internal funds or obtaining new private debt are additional settings in which firms can use their choice of financing to extend the horizon over which they maintain competitive advantage. We leave this to future work.

IV. Innovations in α: Hear no Evil, See no Evil.

The prior sections have dealt with an innovation that provides Firm 1 with an opportunity to profit by increasing s_1. While changes to s_1 must be immediately observable if consumers are going to react, the same does not hold for changes in α. This brings up new strategic possibilities for firms that may wish to hide their new and improved status.

Assume that Firm 1 has an opportunity to immediately raise α_1 to α_1^* at some cost Z which is assumed to be low enough that it pays to innovate. As before, Firm 1 can finance the innovation publicly or privately. For simplicity assume that if it is publicly financed, all information regarding fundamentals (α_1) becomes common knowledge immediately. If the technology is privately financed, Firm 1 can try to "hide" and operate as though it were a low type firm (i.e., it pretends no innovation has occurred) until some finite time T.[31] At time T the true α_1 becomes common knowledge (e.g., through taxes or some other public signal revealed to the market). From there on in both firms play the infinite horizon game knowing the true alpha values.[32]

In the pooling equilibrium focused on here Firm 2 behaves as if Firm 1 is a low type for sure, and Firm 1 behaves as if it is the low type whether or not it truly is. (See the Appendix for a proof that no firm will defect from the proposed equilibrium strategies.) There are two reasons to concentrate on this particular pooling equilibrium: First, under the sufficient condition for its existence given below it is the Pareto dominant equilibrium. Second, if one thinks of the innovation as a true surprise then it is the only Markov equilibrium. Since many innovations are indeed surprises (very low probability events) then this may be the economically most important case to consider when looking at data.

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[31] With some additional algebra (but little additional insight) one can assume that private financing delays the date on which α_1^* becomes common knowledge until date $T^{Private}$ while with public financing the information is revealed at date T^{Public} with $T^{Public} < T^{Private}$.

[32] Unlike the analysis of innovations in s found in Section III.A, this section assumes that firm 2 cannot copy the innovation. It is, however, easy to extend the analysis to case where it can by simply adding a date T_2 at which, for example, α_1^* drops back to α_1. In this case copying does not make the industry more profitable. Rather it just eliminates any competitive advantage one firm has over the other. Other scenarios are also possible.

The potential incentive for the high type to hide comes from the fact that optimal spending (u_1^*) is increasing in α_1. The high type firm would like to credibly commit to spend less on gaining market share (as long as Firm 2 also does so), but cannot do so when there is full information. However, if the conditions for a pooling equilibrium exist then the lack of complete information can make lower spending individually rational. Firm 1 can then continue to hide its new status by spending less on obtaining market share. Pooling will be profitable if the gains from keeping u_1 low outweigh the opportunity cost of a more aggressive campaign. Naturally, the cost is that Firm 1's market share increases at a slower rate. As the Appendix shows a sufficient condition for the pooling equilibrium to exist is that

$$\frac{(\alpha_1^* - \alpha_1)\alpha_1}{(\alpha_1 s_1 + \alpha_2 s_2)} - s_1\left[\frac{\alpha_1^{*3}}{(\alpha_1^* s_1 + \alpha_2 s_2)^2} - \frac{\alpha_1^3}{(\alpha_1 s_1 + \alpha_2 s_2)^2}\right] > 0 \qquad (21)$$

holds. When equation (21) holds a high type Firm 1 is better off under pooling than under separation. Note that it is more likely to hold when α_1^* is small and for very large α_2. Thus, relatively small competitors will choose to hide by adopting the pooling equilibrium strategy. Interestingly the consumer responsiveness parameter (ϕ) does not enter into the conditions under which Firm 1 prefers pooling. This is because the innovation impacts profitability, not s_1, Firm 1's ability to attract customers. Finally, observe that the conditions for the pooling equilibrium hold for any time t. As long as the parameters in the model are such that pooling is preferred, Firm 1 will choose to secretly profit.

Another empirically useful result that comes directly out of equation (21) is that for large enough innovations ($\alpha_1^* \to \infty$) the inequality never holds. Thus, if a firm finds that it can greatly increase its profitability it acts more aggressively right away. Oddly, it is the smaller innovations that firms seek to hide.

An interesting aspect of the pooling equilibrium is that it is *always* preferred by the rival Firm 2. The proof is straightforward and the intuition is as follows: A high type Firm 1 pools by keeping u_1 low. Firm 2 is therefore able to maintain a larger market share while spending less money to compete with Firm 1. In addition, Firm 2's market share during the period of asymmetric information is larger than its equilibrium market share when α_1^* is common knowledge. These two facts mean Firm 2's profits are strictly higher in the pooling than the separating equilibrium.

Given U.S. regulations regarding disclosure for public firms, it can be very difficult for them to withhold information from rivals while complying with the requirement that they keep investors informed. The results in this section show that private financing may provide a mechanism through which firms can commit to less aggressive spending. The intuition developed here can be compared to the capacity pre-commitment in Gelman and Salop (1983) in that by pre-committing (via private financing) to withhold information, an equilibrium outcome is one in which both firms spend less on market share gains. It is also analogous to the "Fat Cat Effect" in Fudenberg and Tirole (1984) in that spending on market share is a strategic complement and private financing provides a commitment device for less aggressive behavior.[33]

V. Relationship to the Prior Literature

IPO activity has been extensively studied in both the theoretical and empirical literature. In their survey paper, Ritter and Welch (2002) provide several reasons for IPOs, including a role for product market competition (such as gaining a first-mover advantage via being the first firm in an industry to have an IPO). Our paper adds to this literature by highlighting that dynamic

[33] Spulber (1995) also shows how, in Bertrand competition, not knowing rivals' costs implies equilibrium prices that are above marginal costs (i.e., information asymmetry softens product market competition).

interactions between rival firms can influence when and if a firm goes public. Ritter and Welch (2002) also note the significant time series variation in IPO activity (e.g., low issuance in the 1980s and high issuance in the 1990s). Shocks to common variables (at the industry or macro-economic level) can be added to our model and would generate patterns in issuance that are consistent with these IPO waves, both in the aggregate economy (through changes in the discount rate) as well as within industries (through changes in industry-level variables). In recent work, Pastor and Veronesi (2005) also link IPO waves to economic fundamentals. They show how decreases in expected returns or increases in future aggregate profitability can cause increases in issuance; however, their focus is on the timing of aggregate IPO activity, not issuance within industries. Further, while a special case of their model might include industry level variables, there is no explicit role for industry competitive dynamics.

This is, of course, not the first paper to posit a relationship between the product and financial markets. However, the previous literature has tended to focus on the strategic use of debt in a firms' capital structure to obtain a competitive advantage. For example, the "limited liability" effect of debt, in which debt commits firms to more aggressive product market behavior is described in Brander and Lewis (1986) and Maksimovic (1988). Bolton and Scharfstein (1990) show how debt can soften competition. Further examples and discussion can be found in the survey by Maksimovic (1990). On the empirical side of the literature, Chevalier (1995) and Leach, Moyen, and Yang (2006) provide evidence on the interaction between leverage and corporate behavior. Chevalier looks at a sample of supermarkets following LBOs and finds that debt "softens" product market competition. In contrast, Leach, Moyen, and Yang look at telecommunications firms and reach the opposite conclusion. Thus, it may be that as yet unmodeled industry characteristics influence the degree to which the predictions in Brander and Lewis (1986) and Maksimovic (1988) are borne out.

MacKay and Phillips (2005) document significant variation in financial structure within industries and that the role of financial structure varies with industry concentration. In particular, in competitive industries, firms with capital-to-labor rations that are close (far away from) to the industry median use less (more) financial leverage. Leverage is higher and more uniform in less competitive industries. In contrast to this literature our paper's focus has been on the public-private decision rather than the leverage decision.

With the exception of Maksimovic and Pichler (2001) in the theoretical literature and recent empirical work Chemmanur, He and Nandy (2007), little has been done to improve our understanding of the potential strategic role played by the private versus public financing decision. Given the size of the private equity (and debt) markets it is important to identify the factors that encourage a firm to use this source of financing rather than the public markets. Indeed, over the past decade or so the private equity market has grown significantly. It is reported to have peaked at $160 billion in 2000, up from $10 billion in 1991 was $40 billion in 2003. (See e.g., "The New Kings of Capitalism," The Economist, 11/27/2004, 373 (8403), 3-5.)

VI. Conclusions

The paper's main goal has been to answer the following questions: First, what are the characteristics of firms that benefit most from innovation and from private financing? Second, how important are industry structure, rival characteristics, and the nature of the innovation? In the context of a dynamic duopoly, we provide closed form solutions for the values of two competing firms, in a setting in which one firm faces an opportunity to innovate. If the technology is adopted, the firm must also determine whether it will obtain public or private financing. Our results relate current firm and industry characteristics to these decision variables. In particular, larger, more profitable firms with small rivals have the greatest incentives to innovate. The private versus public financing decision depends mainly on the magnitude of the technological improvement and length of the period during which private financing extends the innovator's product market advantage. The results from our model suggest that future empirical work examining financing patterns should also explicitly consider competitive dynamics. Calibrations show substantial between and within-industry variation in innovation and financing incentives.

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Appendix

A. The General Model and its Solution

This appendix contains the solution to the most general version of the model discussed in this paper. For the problem described in Section I the value functions for the firms at time 0 are:

$$V_1(m,t) = \int_0^T (\alpha_1 m(t) - u_1 - f_1)e^{-\delta t}dt + B_1 \tag{22}$$

and

$$V_2(m,t) = \int_0^T (\alpha_2(1-m(t)) - u_2 - f_2)e^{-\delta t}dt + B_2 \tag{23}$$

respectively. Here T is a terminal date on which the game ends, and B_i the present value of each firm's value at the terminal date. Note, because the game ends at date T the value functions (V_i) depend both on m and the time remaining until date T.

The analysis seeks a Nash equilibrium in which the players use Markovian strategies. For each firm the instantaneous value functions given by (2) imply that in a Markovian Nash equilibrium the following Hamilton-Jacobi-Bellman (HJB) equations must hold:

$$0 = \max_{u_1} \quad \alpha_1 m - u_1 - f_1 + \frac{\partial V_1}{\partial m}\left\{\frac{\phi[(1-m)s_1u_1 - ms_2u_2]}{u_1s_1 + u_2s_2}\right\} + \frac{\partial V_1}{\partial t} - \delta V_1 \tag{24}$$

and

$$0 = \max_{u_2} \quad \alpha_2(1-m) - u_2 - f_2 + \frac{\partial V_2}{\partial m}\left\{\frac{\phi[(1-m)s_1u_1 - ms_2u_2]}{u_1s_1 + u_2s_2}\right\} + \frac{\partial V_2}{\partial t} - \delta V_2 \tag{25}$$

subject to the terminal condition that $V_i(T)$ equals $B_i(T)$.

The first order condition for Firm 1 is:

$$\frac{\partial V_1}{\partial m}\phi u_2 s_2 s_1 = (u_1s_1 + u_2s_2)^2 \tag{26}$$

and the correspondingly for Firm 2:

$$-\frac{\partial V_2}{\partial m}\phi u_1 s_1 s_2 = (u_1 s_1 + u_2 s_2)^2. \tag{27}$$

Equations (26) and (27) yield equilibrium spending of u_1^* and u_2^*:

$$u_1^* = -\frac{\left(\frac{\partial V_1}{\partial m}\right)^2 \frac{\partial V_2}{\partial m}\phi s_1 s_2}{\left(\frac{\partial V_1}{\partial m}s_1 - \frac{\partial V_2}{\partial m}s_2\right)^2} \tag{28}$$

and

$$u_2^* = \frac{\frac{\partial V_1}{\partial m}\left(\frac{\partial V_2}{\partial m}\right)^2 \phi s_1 s_2}{\left(\frac{\partial V_1}{\partial m}s_1 - \frac{\partial V_2}{\partial m}s_2\right)^2}. \tag{29}$$

Plugging the solutions for u_1^* and u_2^* into the HJB equations (24) and (25) yield the two differential equations (after some extensive algebra):

$$0 = \alpha_1 m - f_1 + \frac{\left(\frac{\partial V_1}{\partial m}\right)^3 \phi s_1^2}{\left(\frac{\partial V_1}{\partial m}s_1 - \frac{\partial V_2}{\partial m}s_2\right)^2} - \frac{\partial V_1}{\partial m}\phi m - \frac{\partial V_1}{\partial t} - \delta V_1 \tag{30}$$

and

$$0 = \alpha_2(1-m) - f_2 - \frac{\left(\frac{\partial V_2}{\partial m}\right)^3 \phi s_2^2}{\left(\frac{\partial V_1}{\partial m}s_1 - \frac{\partial V_2}{\partial m}s_2\right)^2} + \frac{\partial V_2}{\partial m}\phi(1-m) - \frac{\partial V_2}{\partial t} - \delta V_2 \tag{31}$$

that need to be solved.

The solutions for the value functions in (30) and (31) are determined by guessing and verifying that they take on the time dependent forms given by (3) at any date *t*. Those functional forms then imply that the derivatives with respect to *m* and *t* of the value functions equal:

$$\frac{\partial V_1}{\partial m} = -b_1, \qquad \frac{\partial V_1}{\partial t} = -\frac{\partial a_1}{\partial t} - \frac{\partial b_1}{\partial t} m \tag{32}$$

for firm 1 and

$$\frac{\partial V_2}{\partial m} = -b_2, \qquad \frac{\partial V_2}{\partial t} = -\frac{\partial a_2}{\partial t} - \frac{\partial b_2}{\partial t} m \tag{33}$$

for firm 2.

Plugging equations (32) and (33) into (30) and (31) yields a set of differential equations that need to be solved for the a_i and b_i terms subject to the boundary conditions $B_i(T)$. However, since the equalities have to remain true for all m there are in fact four equations that must hold, one for each a_i term and one for each b_i term. After some algebra this implies that solutions need to be found for the following four ordinary differential equations (ODE). For a_1 the ODE is

$$0 = -f_1 + \frac{\phi \alpha_1^3 s_1^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} + \frac{\partial a_1}{\partial t} - \delta a_1 \tag{34}$$

while for a_2 it is

$$0 = \alpha_2 - f_2 + \frac{\phi \alpha_2^3 s_2^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - \frac{\alpha_2}{\phi+\delta} + \frac{\partial a_2}{\partial t} + \delta a_2. \tag{35}$$

Solving for the a_i in the above two equations yields (4) and (5). Next, collect the terms multiplying m to yield for b_1 the ODE:

$$\alpha_1 - \phi b_1 - \frac{\partial b_1}{\partial t} - \delta b_1 = 0 \tag{36}$$

and for b_2:

$$-\alpha_2 - \phi b_2 + \frac{\partial b_2}{\partial t} - \delta b_2 = 0. \tag{37}$$

Solving these last two equations for b_i produces equations (6) and (7).

Given equations (4) through (7) a particular problem's boundary conditions then determine the C_i and k_i terms and thus provide a full characterization of the economy's equilibrium behavior. The main body of the text presents various scenarios, their boundary value conditions, and the solutions they impose on the C_i and k_i terms. There, one can also find the paper's analysis of the economy's overall behavior.

B. Solving for $a_i(t)$ and $b_i(t)$

1. Time Independent Case

In the time independent case the values a_i and b_i do not depend on t. Simple inspection of (4) through (7) then implies that the C_i and k_i terms must equal zero.

2. Time Dependent Case: Value Functions for $t \in [T_1, T_2)$

At time T_2 the value functions given by the time independent case must hold with the s_i set to s_i^* since no further changes in the s_i take place within the model. Based on this one now needs to find solutions for the a_i and b_i as functions of t for the period $t \in [T_1, T_2)$. To do so, the value

functions for the period $t \in [T_2, \infty)$ evaluated at T_2 serve as the boundary conditions:

$$\begin{aligned} a_1(T_2) &= \delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2}\right\} - f_1\right\}; \quad b_1(T_2) = \alpha_1(\phi+\delta)^{-1} \\ a_2(T_2) &= \delta^{-1}\left\{\alpha_2 + (\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_2}{\phi+\delta} - f_2\right\}; \quad b_2(T_2) = -\alpha_2(\phi+\delta)^{-1}. \end{aligned} \tag{38}$$

Given the above boundary conditions, one can solve for the constants C_i and k_i in (4) through (7) to yield the equilibrium value functions:

$$\begin{aligned} V_1(T_1) &= \frac{\alpha_1}{\delta\left(\phi+\delta\right)}\frac{\phi\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} + \\ & e^{-\delta(T_2-T_1)}\frac{\phi\alpha_1}{\delta(\phi+\delta)}\left[\frac{\alpha_1^2 s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_1^2 s_1^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] - \frac{f_1}{\delta} + \frac{\alpha_1}{\phi+\delta} m(T_1) \end{aligned} \tag{39}$$

and

$$V_2(T_1) = \frac{\alpha_2}{\delta\left(\phi+\delta\right)} \frac{\phi\alpha_2^3 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} +$$
$$e^{-\delta(T_2-T_1)} \frac{\phi\alpha_2}{\delta(\phi+\delta)}\left[\frac{\alpha_2^2 s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{\alpha_2^2 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] - \frac{f_2}{\delta} + \frac{\alpha_2}{\phi+\delta}\left(1-m\left(T_1\right)\right). \tag{40}$$

Note that, except for the second terms in each equation, the value functions are of the same form as in the infinite horizon base case. These additional terms can be interpreted as the “first mover advantage” and “second mover disadvantage.” Importantly, the marginal value of market share (and the optimal control) is time independent.

3. Time Dependent Case: Value Functions for t∈[0, T_1)

From the solutions (39) and (40) the boundary value conditions for the b_i are $b_1(T_1) = \alpha_1(\phi+\delta)^{-1}$ and $b_2(T_1) = -\alpha_2(\phi+\delta)^{-1}$ respectively. Plugging these into (36) and (37) yields the result that the k_i terms must equal zero. Thus, $b_1(t) = \alpha_1(\phi+\delta)^{-1}$ and $b_2(t) = -\alpha_2(\phi+\delta)^{-1}$ for all t. Similarly, for the a_i the equilibrium the $C_{i,T1}$ must satisfy:

$$\delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^{*2}}{(\phi+\delta)(\alpha_1 s_1^* + \alpha_2 s_2)^2} - f_1\right] +$$
$$e^{-\delta(T_2-T_1)}(\delta(\phi+\delta))^{-1}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] \tag{41}$$
$$= \delta^{-1}\left[\frac{\phi\alpha_1^3 s_1^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - f_1\right] + C_{1,T1}e^{\delta T_1}$$

and

$$
\begin{aligned}
&\delta^{-1}\left[(\phi+\delta)^{-1}\frac{\phi\alpha_2^3 s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} + \frac{\delta\alpha_2}{\phi+\delta} - f_2\right] + \\
&e^{-\delta(T_2-T_1)}(\delta(\phi+\delta))^{-1}\phi\alpha_2^3\left[\frac{s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right] \\
&= \delta^{-1}\left[\frac{\phi\alpha_2^3 s_2^2}{(\phi+\delta)(\alpha_1 s_1 + \alpha_2 s_2)^2} - f_2\right] + C_{2,T1}e^{\delta T_1}.
\end{aligned} \tag{42}
$$

Solving for $C_{1,\mathrm{T}1}$ and $C_{2,T1}$ produces:

$$
\begin{aligned}
C_{1,T_1} &= (\delta(\phi+\delta))^{-1}e^{-\delta T_1}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{s_1^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2}\right] \\
&+(\delta(\phi+\delta))^{-1}e^{-\delta T_2}\phi\alpha_1^3\left[\frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_1^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right]
\end{aligned} \tag{43}
$$

and

$$
\begin{aligned}
C_{2,T_1} &= (\delta(\phi+\delta))^{-1}e^{-\delta T_1}\phi\alpha_2^3\left[\frac{s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2} - \frac{s_2^2}{(\alpha_1 s_1 + \alpha_2 s_2)^2}\right] \\
&+e^{-\delta T_2}(\delta(\phi+\delta))^{-1}\phi\alpha_2^3\left[\frac{s_2^{*2}}{(\alpha_1 s_1^* + \alpha_2 s_2^*)^2} - \frac{s_2^2}{(\alpha_1 s_1^* + \alpha_2 s_2)^2}\right].
\end{aligned} \tag{44}
$$

C. Comparative Statics: Private versus Public Financing

1. $\partial\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right)/\partial\alpha_1$

Equation (18) characterizes Firm 1's incentives to finance publicly as we vary its revenue-generating ability: $\dfrac{\partial(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_1} = q\alpha_1^2\left[\dfrac{\psi^{*2}(\alpha_1\psi^* + 3\alpha_2)}{(\alpha_1\psi^* + \alpha_2)^3} - \dfrac{\psi^2(\alpha_1\psi + 3\alpha_2)}{(\alpha_1\psi + \alpha_2)^3}\right].$

To sign this, note that at $\psi^*=\psi$, (18) equals 0. Since $\dfrac{\partial\left(\dfrac{\alpha_1\psi^3+3\alpha_2\psi^2}{(\alpha_1\psi+\alpha_2)^3}\right)}{\partial\psi}=(\alpha_1\psi+\alpha_2)^{-4}6\alpha_2^2\psi$ is

clearly positive, then for all $\psi^*>\psi$, $\dfrac{\psi^{*2}(\alpha_1\psi^*+3\alpha_2)}{(\alpha_1\psi^*+\alpha_2)^3}>\dfrac{\psi^2(\alpha_1\psi+3\alpha_2)}{(\alpha_1\psi+\alpha_2)^3}$. Thus, (18) is positive and the incentive to secure private financing for an innovation is increasing in Firm 1's revenue generating ability (α_1).

2. $\partial\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\alpha_2$

Equation (19) shows how Firm 1's incentives to finance publicly vary with the profitability of Firm 2: $\dfrac{\partial(V_1^{\text{Private}}-V_1^{\text{Public}})}{\partial\alpha_2}=2q\alpha_1^3\left[\dfrac{\psi^2}{(\alpha_1\psi+\alpha_2)^3}-\dfrac{\psi^{*2}}{(\alpha_1\psi^*+\alpha_2)^3}\right]$. Observe that at $\alpha_2=0$,

Equation (19) is greater than zero since $\psi^*>\psi\Rightarrow\dfrac{1}{\psi}-\dfrac{1}{\psi^*}>0$. Also note that

$\dfrac{\partial\left(\dfrac{\psi^2}{(\alpha_1\psi+\alpha_2)^3}\right)}{\partial\psi}=(\alpha_1\psi+\alpha_2)^{-4}\left(2\alpha_2\psi-\alpha_1\psi^2\right)$. This means that when $\alpha_2>(\alpha_1\psi/2)$, Equation (19) is negative. Firm 1's incentives to obtain private financing are decreasing in α_2 when α_2 is large relative to α_1. The opposite is true for small α_2.

3. $\partial\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\psi$

The impact of the size of Firm 1's relative spending advantage (ψ parameter) on incentives to finance publicly is also of interest. Since $\dfrac{\partial\left(\dfrac{\psi}{(\alpha_1\psi+\alpha_2)}\right)}{\partial\psi}=(\alpha_1\psi+\alpha_2)^{-2}\alpha_2>0$,

Equation (17) implies that increasing Firm 1's spending advantage due to the technology (high

ψ^*) will increase incentives to finance publicly. Conversely, increasing Firm 1's current spending advantage (ψ) decreases its incentive to finance innovations with privately issued securities.

4. $\partial\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right) / \partial\phi$

Since $\partial q/\partial\phi>0$ and since ϕ does not enter (17) outside of q one has $\partial\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right) / \partial\phi>0$.

5. $\partial^2\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right) / \partial\alpha_1\partial\alpha_2$

We now consider the impact of an increase in the profitability of a rival on the value of an increase in an innovator's profitability (and incentive to remain private):

$$\frac{\partial^2(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_1\partial\alpha_2} = 6q\alpha_1^2\alpha_2\left[\frac{\psi^2}{(\alpha_1\psi + \alpha_2)^4} - \frac{\psi^{*2}}{(\alpha_1\psi^* + \alpha_2)^4}\right].$$ Since

$$\frac{\partial\left(\dfrac{\psi^2}{(\alpha_1\psi + \alpha_2)^4}\right)}{\partial\psi} = (\alpha_1\psi + \alpha_2)^{-5}2\psi(\alpha_2 - \psi\alpha_1)$$ is greater than zero when α_1 is small (i.e., $\alpha_{2}/\psi > \alpha_1$), this implies that increasing the profitability of a rival increases the value of a profitability increase for less profitable innovators.

6. $\partial^2\left(V_1^{\text{Private}} - V_1^{\text{Public}}\right) / \partial\alpha_1\partial\delta$

We now consider the impact of an increase discount rate on the value of an increase in an innovator's profitability (and its impact on incentive to remain private):

$$\frac{\partial^2(V_1^{\text{Private}} - V_1^{\text{Public}})}{\partial\alpha_1\partial\delta} = -(\phi + 2\delta)(\delta(\phi+\delta))^{-2}\left[e^{-\delta T_2^{\text{Public}}} - e^{-\delta T_2^{\text{Private}}}\right]\phi\alpha_1^2\left[\frac{\psi_2^2(\alpha_1\psi^* + 3\alpha_2)}{(\alpha_1\psi^* + \alpha_2)^3} - \frac{\psi_1^2(\alpha_1\psi + 3\alpha_2)}{(\alpha_1\psi + \alpha_2)^3}\right]$$

As we showed in Case 1 above, the last term is positive, so this expression is negative. An increase in the discount rate decreases the impact of an increase in the innovator's profitability on its incentive to remain private.

7. $\partial^2\left(V_1^{\text{Private}}-V_1^{\text{Public}}\right)/\partial\alpha_2\partial\delta$

Similarly, consider the impact of an increase discount rate on the value of an increase in a rival's profitability (and its impact on the innovator's incentive to remain private):

$$\frac{\partial^2(V_1^{\text{Private}}-V_1^{\text{Public}})}{\partial\alpha_2\partial\delta}$$

$$=-2(\phi+2\delta)(\delta(\phi+\delta))^{-2}\left[e^{-\delta T_2^{\text{Public}}}-e^{-\delta T_2^{\text{Private}}}\right]\phi\alpha_1^3\left[\frac{\psi^2}{(\alpha_1\psi+\alpha_2)^3}-\frac{\psi^{*2}}{(\alpha_1\psi^*+\alpha_2)^3}\right]$$

This sign depends on the relative values of α_1 and α_2 as in Case 2 above.

D. Changes in α: Pooling versus Separating

Proposition: A Pareto optimal pooling equilibrium exists whenever equation (21) holds. In this pooling equilibrium the following conditions hold: (1) The low type Firm 1 optimizes as if it is in the full information case. (2) The high type Firm 1 mimics the low type's actions along all game paths. (3) Firm 2 optimizes as if Firm 1 is a low type for sure. (4) If Firm 1 deviates from the equilibrium path then Firm 2 plays as if Firm 1 is a high type for sure.

Proof: As will be shown the equilibrium holds up under the standard refinements, properly interpreted for the model's continuous time setting. It is easiest to begin with Firm 2. Consider some non-degenerate prior over Firm 1's type. In the pooling equilibrium Firm 2 believes its value function (3) at time T will equal $V_2\left(m,T\middle|\alpha_1\right)$ or $V_2\left(m,T\middle|\alpha_1^*\right)$ for the infinite horizon case with probabilities based on its priors. Note, that in either case the solution to b_2 for $t\geq T$ is identical since b_2 does not depend on α_1. Thus, the terminal condition for the differential equation yielding b_2 for $t<T$ is independent of Firm 1's realized type. Given that Firm 2 believes Firm 1 will play as a low type firm regardless of Firm 1's true type the HJB equation (25) must continue to hold for $t<T$. From here it is simple to show that the solution to b_2 for $t<T$ is unchanged from the full information case. Thus, for Firm 2, all that changes in equilibrium is the constant a_2 which does not influence its behavior. Since Firm 2 was optimizing in the full information case it must remain optimal to follow the same strategy in the pooling equilibrium. The only remaining issue for Firm 2 is whether it proposed out of equilibrium beliefs are "credible." To answer this question turn now to the incentives for each Firm 1 type.

Proving the low type Firm 1 does not wish to defect from the equilibrium is trivial. Under the pooling equilibrium the low type receives the same profits as it would under full revelation. Thus, the only question is whether a defection that leads Firm 2 to believe Firm 1 is a high type can make the low type better off. In the candidate equilibrium a defection causes Firm 2 to act as though Firm 1 is a high type for sure. Firm 2's equilibrium spending levels on market share will therefore be based on the solution to (25) with $\alpha_1 = \alpha_1^*$. Some algebra shows that this just leads to higher spending on market share acquisition by Firm 2 which can only make Firm 1 worse off.

The final player is the high type Firm 1. The Nash equilibrium of the full information game at the time of the improvement in α (time 0) is

$$V_1^{FullInfo}(0) = \delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^{*3}s_1^2}{(\alpha_1^* s_1 + \alpha_2 s_2)^2}\right\} - f_1\right\} + (\phi+\delta)^{-1}\alpha_1^* m(0). \qquad (45)$$

Consider now the value function when Firm 1 decides to privately finance in order to hide its true revenue generating capacity until T. In this case, Firm 1 obtains the profits of the low-type firm, plus $\left(\alpha_1^* - \alpha_1\right)m(t)$ at each instant between 0 and *T*. If it pools, the value function for Firm 1 is:

$$
\begin{aligned}
V_1^{Pool}(0) = {} & \delta^{-1}\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right\}-f_1\right\}+(\phi+\delta)^{-1}\alpha_1 m(0) \\
& -e^{-\delta T}\left[\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^3 s_1^2}{(\alpha_1 s_1+\alpha_2 s_2)^2}\right\}-f_1\right\}+(\phi+\delta)^{-1}\alpha_1 m(T)\right] \\
& +e^{-\delta T}\left[\left\{(\phi+\delta)^{-1}\left\{\frac{\phi\alpha_1^{*3} s_1^2}{(\alpha_1^* s_1+\alpha_2 s_2)^2}\right\}-f_1\right\}+(\phi+\delta)^{-1}\alpha_1^* m(T)\right] \\
& +\int_0^T e^{-\delta t}(\alpha_1^*-\alpha_1)m(t)dt
\end{aligned}
\tag{46}
$$

The first two terms of (46) represent the full-information profits of a low-type firm from 0 to T, the third term is the full-information profit of a high-type firm from T to ∞ and the last term represents the discounted profits associated with a high-type firm hiding and pretending to be a low-type firm.

To find the conditions under which $V_1^{Pool} - V_1^{FullInfo} > 0$ the first step is to find a solution for the last term in (46). Note that, while hiding, equilibrium spending is given by (8) with the values of alpha given by α_1 and α_2. Therefore, *dm* must equal:

$$dm = \frac{\phi s_1 \alpha_1}{s_1\alpha_1 + s_2\alpha_2} - \phi m. \tag{47}$$

Solving this ordinary differential equation for m(t) yields $m(t) = Ce^{-\phi t} + \frac{s_1\alpha_1}{s_1\alpha_1 + s_2\alpha_2}$ where *C* is a constant whose solution is given by the boundary value. At t=0,

$m(0) = C + \frac{s_1\alpha_1}{s_1\alpha_1 + s_2\alpha_2} \Rightarrow C = m(0) - \frac{s_1\alpha_1}{s_1\alpha_1 + s_2\alpha_2}$. The general solution is:

$$m(t) = \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2} + \mathrm{e}^{-\delta t}\left[m(0) - \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}\right]. \tag{48}$$

Substitution into $\int_0^T e^{-\delta t}(\alpha_1^* - \alpha_1)m(t)dt$ gives:

$$\begin{aligned}
&\int_0^T e^{-\delta t}(\alpha_1^* - \alpha_1)\left[\frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2} + e^{-\phi t}\left[m(0) - \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}\right]\right] dt = \\
&(\alpha_1^* - \alpha_1)\left[(1 - e^{-\delta T})\delta^{-1}\frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2} + (1 - e^{-(\delta+\phi)T})(\delta+\phi)^{-1}\left(m(0) - \frac{\alpha_1 s_1}{\alpha_1 s_1 + \alpha_2 s_2}\right)\right].
\end{aligned} \tag{49}$$

Plugging this into the pooling value function (46) and subtracting $V_1^{FullInfo}$ implies that pooling is preferred to separation when (21) holds. Thus, when (21) holds any defection that leads Firm 2 to believe Firm 1 is a high type firm must make Firm 1 worse off. Given the conjectured equilibrium strategies and beliefs it follows that a high type Firm 1 will not defect from the proposed equilibrium.

The last issue is whether the proposed out of equilibrium beliefs for Firm 2 are credible under the usual refinements. If Firm 2 sees a defection the set of best responses lie within those that correspond to the actions it would take under some equilibrium with the belief that Firm 1 is a high type with a nonzero probability. It is easy to show that any such best response involves higher spending on u_2 which makes both the high and low Firm 1 types worse off. Thus when (21) holds, there does not exist a best response by Firm 2 that can make either Firm 1 type better off. Under the standard refinements Firm 2 can then act as though it believes Firm 1 is a high type for sure thereafter. QED

Table 1: Change in the Value to the Innovator from an Innovation

Derivative	Economic Interpretation	Sign	Condition
$\partial^2 V_1(0)/\partial\psi^* d\alpha_1$	The impact of an increase in the innovator's profitability on the value of any innovation.	+	All firms.
$\partial^2 V_1(0)/\partial\psi^* d\alpha_2$	The impact of an increase in the rival's profitability on the value of any innovation.	-	Small firms.
$\partial^2 V_1(0)/\partial\psi^* \partial\alpha_2$	The impact of an increase in the rival's profitability on the value of any innovation.	+	Large firms.
$\partial^2 V_1(0)/\partial\psi^* \partial\delta$	An increase in the discount rate reduces the value of any innovation.	-	All firms.
$\partial^2 V_1(0)/\partial\psi^* \partial\phi$	An increase in consumer responsiveness increases the value of any innovation.	+	All firms.

Table 2: Possible Empirical Proxies for the Model's Parameters		
Parameter	Description	Possible Empirical Proxies
m	Market share	Share of total industry: ▪ Sales ▪ Assets ▪ Market Value Equity + Book Vale of Total Debt
u	Spending to gain market share	▪ Advertising ▪ R&D ▪ Capital Expenditures
ϕ, s	Effectiveness of spending, and consumer responsiveness.	▪ Estimation based on the discrete time version of equation (1) : $m_{t+1} - m_t = \frac{\phi u_1 s_1}{u_1 s_1 + u_2 s_2} - \phi m_t$
f	Costs of Operations	▪ Operating Expenses (net of proxy for market share spending)
α	Revenue-generating ability	▪ Sales ▪ Operating Profit
δ	r-g: discount rate minus industry growth rate	▪ Interest Rates ▪ Industry Growth Rate
T_1, T_2		▪ Patent protection periods ▪ R&D expenditures
Other Variables of Interest		
	Opportunities to Innovate	▪ Number of Patents ▪ R&D Expenditures
$\alpha_1 - \alpha_2$	Relative competitive advantage	▪ Difference in market shares ▪ Industry HHI

Table 3: Change in the Value of Remaining Private ($V_1^{\text{Private}} - V_1^{\text{Public}}$) when Financing an Innovation

Derivative w.r.t.	Economic Interpretation	Sign	Condition
α_1	The impact of an increase in the innovator's profitability.	+	All firms.
α_2	The impact of an increase in the rival's profitability.	-	Small innovator.
α_2	The impact of an increase in the rival's profitability.	+	Large innovator.
Ψ	The relative ability of the innovator to take market share relative to its rival.	-	All firms.
Ψ^*	The innovation's improvement in the innovator's ability to increase its market share during the period it has a technological advantage, T_1 to T_2.	+	All firms.
ϕ	Consumer responsiveness to corporate spending seeking to increase market share.	+	All firms.
δ	An increase in the real interest rate.	-	All firms.
T_2^{Public}	Increase in the time during which the innovating firm can maintain its advantage relative to the rival in a full-disclosure setting.	-	All firms.
T_2^{Private}	Increase in the time during which the innovating firm can maintain its advantage relative to the rival when private financing is chosen.	+	All firms.

Table 4: Estimated Market Share Half Lives By Four Digit SIC Industry

The ϕ parameter is estimated for each individual 4 digit SIC industry (as classified by COMPUSTAT) using equation (20) – a discrete time version of the law of motion for market share. The estimated value of ϕ is the one that minimizes the sum of squared errors, ε_i. Firm i's market share, $m_{i,t}$ is defined as the share of sales of all *CRSP/COMPUSTAT* firms in the industry. The values of u_i are taken to be either capital expenditures or the sum of capital expenditures, research and development and advertising (based on the lowest sum of squared errors). The half life listed uses the point estimates for ϕ. Based on equation (1) the half life (h) equals $\ln(2)/\phi$. This half life represents, in years, the time it would take a firm that spends nothing on customer recruiting to lose half its current market share.

SIC CODE	Industry Name	Est. φ	Std. Err.	Half Life (h)
800	Forestry	0.671	0.160	1.03
1221	Bitmns Coal,Lignite Surf Mng	0.166	0.175	4.19
1731	Electrical Work	0.542	0.058	1.28
2033	Can Fruit,Veg,Presrv,Jam,Jel	0.240	0.031	2.89
2273	Carpets And Rugs	0.131	0.036	5.28
2390	Misc Fabricated Textile Pds	0.093	0.104	7.43
2451	Mobile Homes	0.216	0.052	3.22
2510	Household Furniture	0.204	0.027	3.39
2520	Office Furniture	0.533	0.149	1.30
2650	Paperboard Containers, Boxes	0.276	0.034	2.51
2741	Miscellaneous Publishing	0.383	0.058	1.81
2771	Greeting Cards	0.163	0.067	4.25
2800	Chemicals & Allied Prods	0.360	0.039	1.93
2842	Special Clean,Polish Preps	0.402	0.042	1.73
2890	Misc Chemical Products	0.391	0.037	1.77
3011	Tires And Inner Tubes	0.382	0.135	1.81
3060	Fabricated Rubber Pds, Nec	0.196	0.018	3.53
3086	Plastics Foam Products	0.488	0.075	1.42
3241	Cement, Hydraulic	0.248	0.106	2.80
3270	Concrete, Gypsum And Plaster	0.226	0.023	3.07
3272	Concrete Pds, Ex Block,Brick	0.072	0.050	9.61
3320	Iron And Steel Foundries	0.341	0.063	2.03
3470	Coating,Engraving,Allied Svc	0.076	0.033	9.10
3530	Constr,Mining,Matl Handle Eq	0.424	0.042	1.63
3541	Machine Tools, Metal Cutting	0.226	0.049	3.07
3550	Special Industry Machinery	0.484	0.051	1.43
3561	Pumps And Pumping Equipment	0.230	0.044	3.02
3575	Computer Terminals	0.593	0.069	1.17
3590	Misc Indl, Coml, Machy & Eq	0.326	0.050	2.13
3670	Electronic Comp, Accessories	0.318	0.042	2.18
3713	Truck And Bus Bodies	0.144	0.034	4.81
3751	Motorcycles,Bicycles & Parts	0.659	0.094	1.05
3821	Lab Apparatus And Furniture	0.263	0.045	2.63
3829	Meas & Controlling Dev, Nec	0.347	0.043	2.00
3942	Dolls And Stuffed Toys	0.202	0.076	3.43
3960	Costume Jewlry,Button,Notion	0.435	0.167	1.59
5010	Motor Veh Parts, Supply-Whsl	0.178	0.036	3.89
5070	Hardwr, Plumb, Heat Eq-Whsl	0.234	0.053	2.96
5072	Hardware-Wholesale	0.161	0.034	4.31
5130	Apparel,Piece Gds,Notns-Whsl	0.412	0.072	1.68

Table 4: Estimated Market Share Half Lives By Four Digit SIC Industry

The ϕ parameter is estimated for each individual 4 digit SIC industry (as classified by COMPUSTAT) using equation (20) – a discrete time version of the law of motion for market share. The estimated value of ϕ is the one that minimizes the sum of squared errors, ε_i. Firm i's market share, $m_{i,t}$ is defined as the share of sales of all *CRSP/COMPUSTAT* firms in the industry. The values of u_i are taken to be either capital expenditures or the sum of capital expenditures, research and development and advertising (based on the lowest sum of squared errors). The half life listed uses the point estimates for ϕ. Based on equation (1) the half life (h) equals $\ln(2)/\phi$. This half life represents, in years, the time it would take a firm that spends nothing on customer recruiting to lose half its current market share.

SIC CODE	Industry Name	Est. φ	Std. Err.	Half Life (h)
5141	Groceries, General Line-Whsl	0.250	0.048	2.77
5171	Petroleum Bulk Stations-Whsl	0.071	0.039	9.76
5412	Convenience Stores	0.323	0.033	2.15
5531	Auto And Home Supply Stores	0.112	0.014	6.17
5900	Miscellaneous Retail	0.135	0.030	5.14
5912	Drug & Proprietary Stores	0.181	0.028	3.83
7310	Advertising	0.121	0.020	5.71
7380	Misc Business Services	0.587	0.016	1.18
7381	Detect,Guard,Armor Car Svcs	0.298	0.057	2.33
7830	Motion Picture Theaters	0.125	0.016	5.57
7996	Amusement Parks	0.436	0.064	1.59
8000	Health Services	0.260	0.029	2.66
8050	Nursing & Personal Care Fac	0.396	0.129	1.75
8090	Misc Health & Allied Svc,Nec	0.319	0.032	2.17
8734	Testing Laboratories	0.308	0.041	2.25

Table 5
Selected Industries: Estimated Consumer Responsiveness, Spending Effectiveness and Profitability
The φ and *s* parameters are estimated for each individual 4 digit SIC industry using a discrete time version of the law of motion for market share (Equation 1). Firm i's market share, $m_{i,t}$ is defined as the share of sales of all CRSP/COMPUSTAT firms in the industry. The values of u_i are taken to be the sum of capital expenditures, research and development and advertising. The individual firm profitability parameters (α_i) are estimated via OLS, based on Equation (2) in the text.

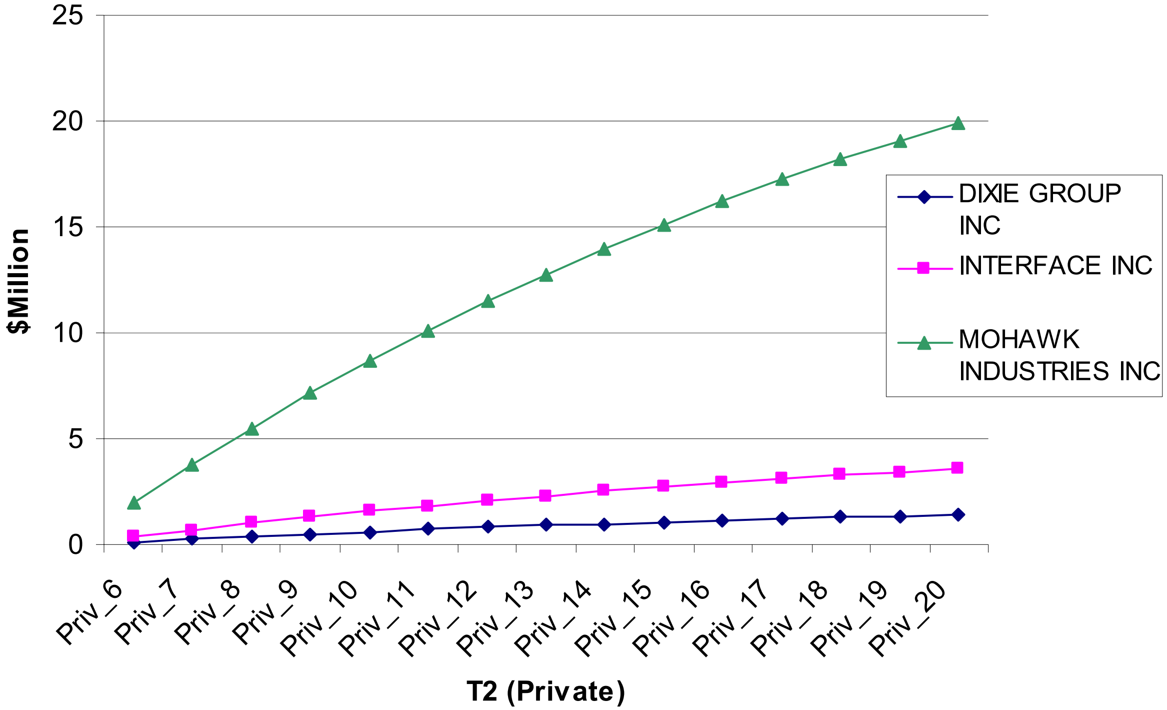
SIC	Industry Name	Company	Est. φ	φ_se	Est. *s*	*s_se*	Est. α	α_se
2273	Carpets And Rugs	Dixie Group Inc	0.131	0.036	0.039	0.037	86.7	21.8
2273	Carpets And Rugs	Carriage Industries Inc	0.131	0.036	0.159	0.263	-66.1	97.5
2273	Carpets And Rugs	Horizon Industries Inc	0.131	0.036	0.057	0.213	211.7	133.3
2273	Carpets And Rugs	Interface Inc	0.131	0.036	0.136	0.063	61.5	20.2
2273	Carpets And Rugs	Shaw Industries Inc	0.131	0.036	0.155	0.061	55.4	25.2
2273	Carpets And Rugs	Salem Carpet Mills Inc	0.131	0.036	0.054	0.239	228.0	89.6
2273	Carpets And Rugs	Mohawk Industries Inc	0.131	0.036	0.401	0.183	97.8	4.6
3086	Plastics Foam Products	Radva Corp	0.488	0.075	0.175	0.048	1.9	1.5
3086	Plastics Foam Products	Tuscarora Inc	0.488	0.075	0.057	0.007	5.9	2.4
3086	Plastics Foam Products	Norwood Promotional Products Inc	0.488	0.075	0.295	0.043	2.2	0.1
3086	Plastics Foam Products	Advanced Materials Group Inc	0.488	0.075	0.203	0.059	2.0	1.1
3086	Plastics Foam Products	Foamex International Inc	0.488	0.075	0.194	0.017	1.1	2.3
3942	Dolls And Stuffed Toys	Middleton Doll Company	0.202	0.076	0.097	0.203	738.4	123.5
3942	Dolls And Stuffed Toys	Mattel Inc	0.202	0.076	0.041	0.015	632.1	34.3
3942	Dolls And Stuffed Toys	Russ Berrie & Co	0.202	0.076	0.189	0.088	558.5	87.0
3942	Dolls And Stuffed Toys	Vermont Teddy Bear Company The	0.202	0.076	0.045	0.120	565.6	53.2
3942	Dolls And Stuffed Toys	Play By Play Toys & Nvlts Inc	0.202	0.076	0.201	0.218	216.5	130.7
3942	Dolls And Stuffed Toys	Jakks Pacific Inc	0.202	0.076	0.228	0.114	298.2	38.6
3942	Dolls And Stuffed Toys	Boyds Collection Ltd	0.202	0.076	0.199	0.244	1215.9	143.4
7381	Detect,Guard,Armor Car Svcs	Brinks Co	0.298	0.057	0.026	0.011	28.7	6.0
7381	Detect,Guard,Armor Car Svcs	Compudyne Corp	0.298	0.057	0.019	0.039	19.6	8.2
7381	Detect,Guard,Armor Car Svcs	Wackenhut Corp	0.298	0.057	0.162	0.071	13.0	2.3
7381	Detect,Guard,Armor Car Svcs	Pinkertons Inc New	0.298	0.057	0.273	0.092	0.2	6.5
7381	Detect,Guard,Armor Car Svcs	Command Security Corp	0.298	0.057	0.430	0.203	18.6	9.7
7381	Detect,Guard,Armor Car Svcs	Burns International Svcs Corp	0.298	0.057	0.040	0.019	99.6	18.6
7381	Detect,Guard,Armor Car Svcs	Kroll Inc	0.298	0.057	0.051	0.044	161.0	25.5

Table 6
Calibrations: Estimated Value of Obtaining Private Financing in Four Industries (opportunity to increase *s* by 50%)

This table presents the mean difference in the values of firms financed publicly versus privately for two industries, given an opportunity to increase spending effectiveness parameter s_i by 50% in the year 2005. This is $\Delta V = V_{private} - V_{public}$, as specified in (17). *ZD* is set to zero, for straightforward comparisons. (An alternative would be to apply a 20% private market discount to the cost of the innovation. See Hertzel and Smith (1993)). Within each industry, firm-specific α's and s's and were estimated based on equations (2) and (9), respectively. In estimating ΔV for firm i, s_2 is set to the market-share-weighted average of all competing firm s_j and α_2 is the market-share weighted mean α of all competing firms in the industry during the year 2005. All firms for which we were able to obtain estimates of α and s are included.

Industry Name	Carpets and Rugs	Plastics Foam Products	Dolls and Stuffed Toys	Detect, Guard, Armor Car Services
SIC Code	2273	3086	3942	7381
ϕ	0.13	0.49	0.20	0.30
Mean Value $Mil 2005 from Private Financing:				
2 additional years of competitive advantage	1.57	0.04	33.91	1.05
5 additional years of competitive advantage	3.61	0.09	79.70	2.35
10 additional years of competitive advantage	6.31	0.18	144.26	3.93
15 additional years of competitive advantage	8.32	0.25	196.54	4.99

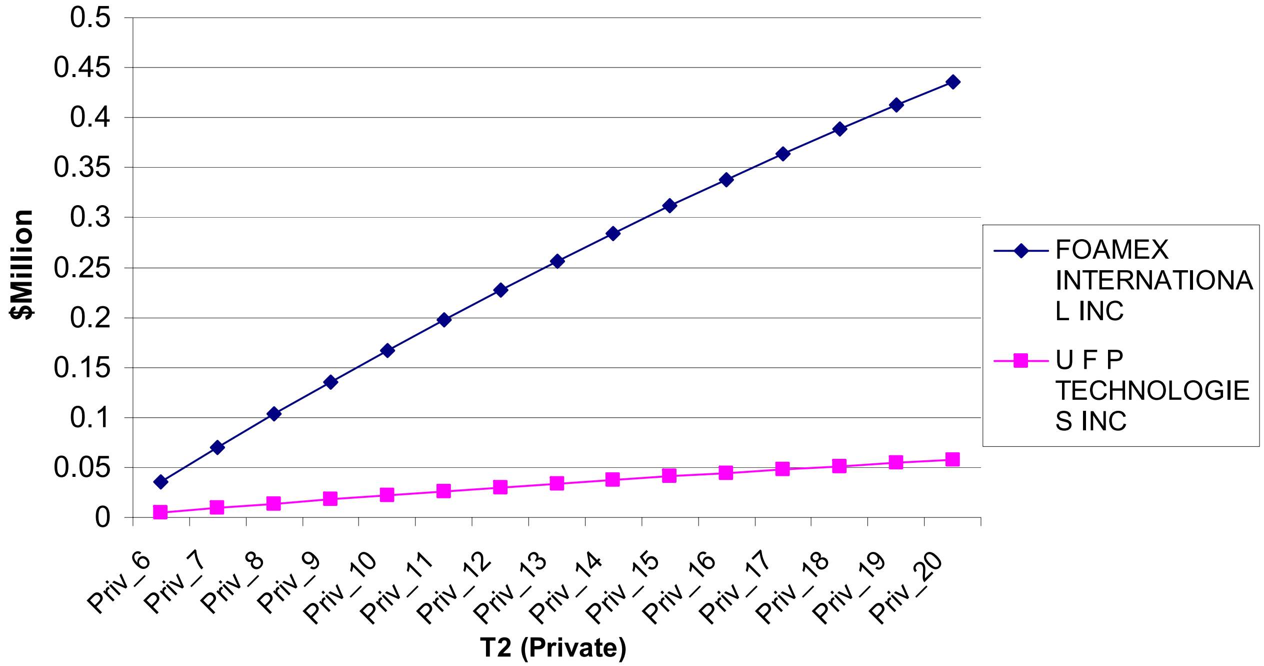
Figure 2a
Carpets and Rugs Industry, 2005: Value of Remaining Private, Given An Innovation offering a 50% Increase in *s*



This figure presents results from a calibration that estimates each firm's value change given an opportunity to innovate that results in a 50% increase in spending effectiveness (s) for three firms in industry SIC 2273 for which 2005 sales and spending data are in COMPUSTAT. The parameters for the firms and industries are as follows:

PERMNO	Company	s	α	φ	Market Share
10886	Dixie Group Inc	0.039	86.70	0.131	0.040
44768	Interface Inc	0.136	61.46	0.131	0.124
77496	Mohaw k Industries	0.401	97.77	0.131	0.835

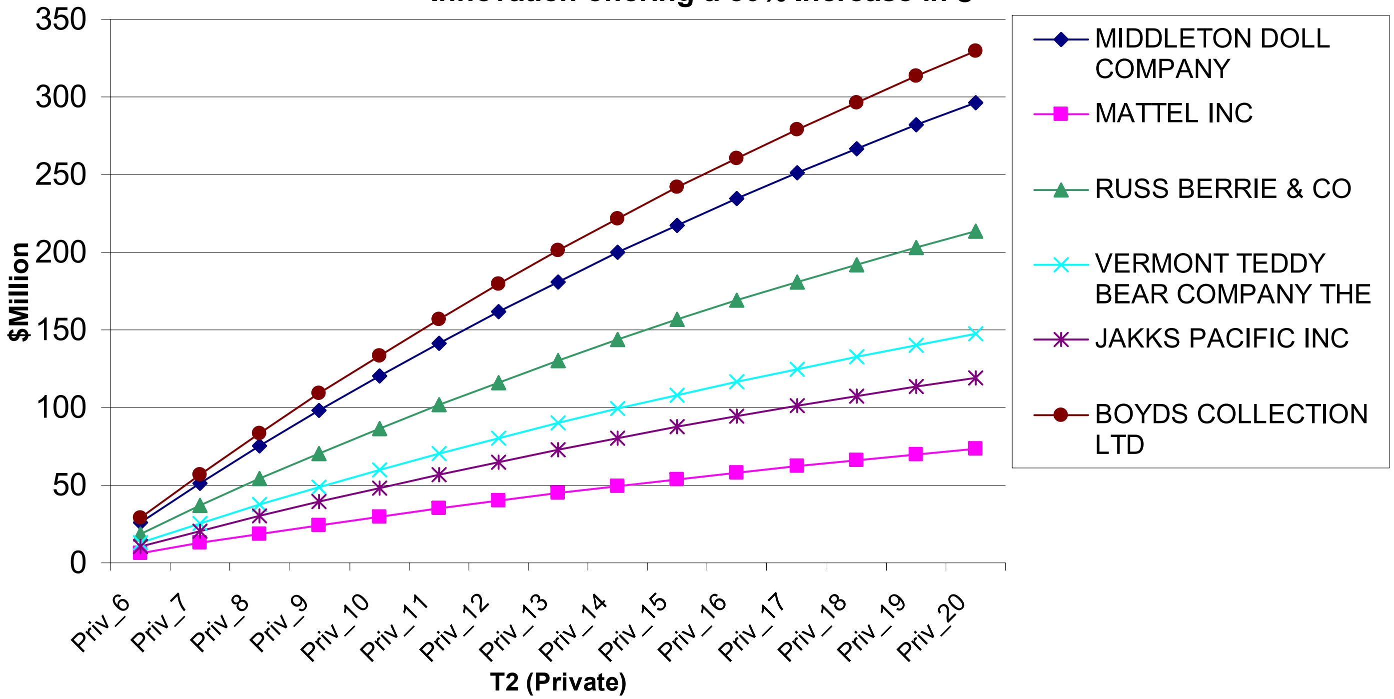
Figure 2b
Plastics Foam Products Industry, 2005: Value of Remaining Private ($Mil), Given An Innovation offering a 50% Increase in *s*



This figure presents results from a calibration that estimates each firm's value change given an opportunity to innovate that results in a 50% increase in spending effectiveness (s) for two firms in industry SIC 3086 for which 2005 sales and spending data are in COMPUSTAT. The parameters for the firms and industries are as follows:

PERMNO	Company	s	α	φ	Market Share
79971	Foamex International Inc	0.194	1.07	0.488	0.940
80050	U F P Technologies Inc	0.077	0.57 3	0.488	0.060

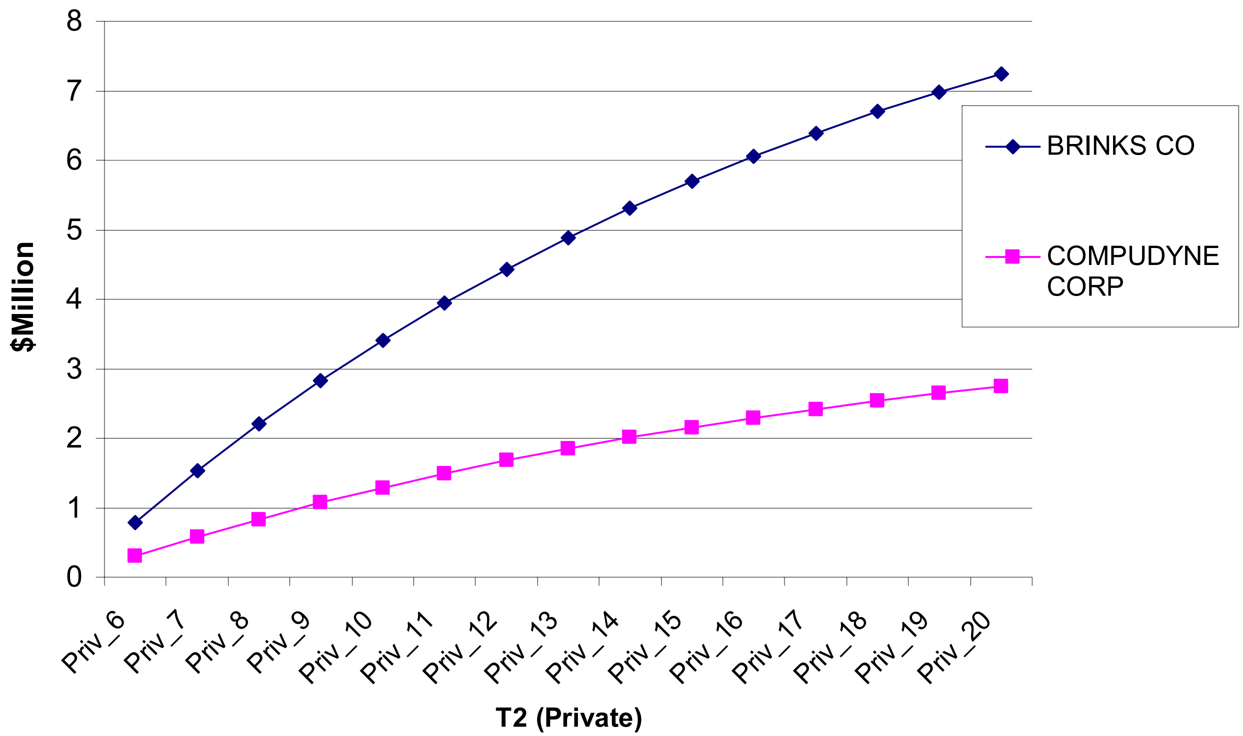
Figure 2c
Dolls and Stuffed Toys Industry, 2005: Value of Remaining Private, Given An Innovation offering a 50% Increase in *s*



This figure presents results from a calibration that estimates each firm's value change given an opportunity to innovate that results in a 20% increase in spending effectiveness (s) for the six firms in industry SIC 3942 for which 2005 sales and spending data are in COMPUSTAT. The parameters for the firms and industries are as follows:

PERMNO	Company	s	α	φ	Market Share
11318	Middleton Doll Company	0.097	738.41	0.202	0.003
39538	Mattel Inc	0.041	632.09	0.202	0.823
66050	Russ Berrie & Co	0.189	558.53	0.202	0.046
79801	Vermont Teddy Bear Co.	0.045	565.59	0.202	0.011
83520	Jakks Pacific Inc	0.228	298.15	0.202	0.105
86748	Boyds Collection Ltd	0.199	1215.94	0.202	0.012

Figure 2d
Detect, Guard, Armor Car Industry: Value of Remaining Private ($Mil), Given An Innovation offering a 50% Increase in *s*



This figure presents results from a calibration that estimates each firm's value change given an opportunity to innovate that results in a 50% increase in spending effectiveness (s) for four firms in industry SIC 7381 for which 2005 sales and spending data are in COMPUSTAT. The parameters for the firms and industries are as follows:

PERMNO	Company	s	α	φ	Market Share
18649	Brinks Co	0.026	28.74	0.298	0.947
30744	Compudyne Corp	0.019	19.65	0.298	0.053